Technical Briefing: Scour Criteria and Stable Channel Design

Executive Summary

This briefing examines the physical foundations of incipient motion and their application to the design of stable hydraulic channels. Incipient motion occurs when the boundary shear stress applied by fluid flow equals the critical shear stress of the sediment:

$$\tau_0 = \tau_c$$

The document synthesizes the mechanics of particle-level forces, the dimensionless thresholds defined by the Shields and Coleman–Ikeda–Iwagaki (C–I–I) models, and the geometric requirements for stable channels.

Key Takeaways

- **Threshold Mechanics:** The onset of scour is governed by the balance of hydrodynamic forces (drag and lift) against resisting forces (submerged weight and friction).
- **The C–I–I Model:** Bridges grain-scale physics to channel-scale design using particle-level velocity U_p across viscous, transitional, and rough flow regimes.
- **Side-Slope Sensitivity:** Banks are inherently less stable than the channel bed. Stability reduction factors K must be applied, with lift-inclusive models providing more conservative design.
- **Glover–Florey–Lane (G–F–L) Theory:** Ideal stable channels place every grain along the wetted perimeter simultaneously at the threshold of motion, producing a cosine-shaped cross-section.

1. Foundations of Incipient Motion

Incipient motion focuses on the mechanics of **initial sediment movement**, not transport rate.

Assumptions

- Non-cohesive sediment
- Uniform particle diameter d
- Steady, uniform flow
- Clear-water threshold

- No vegetation

1.1 Particle Mechanics and Angle of Repose

Submerged weight:

$$W' = (\gamma_s - \gamma)V$$

Internal friction coefficient:

$$\mu = \tan\ \phi$$

Typical angles of repose

- Sand: 28–34°
- Gravel: up to 40°
- Crushed ledge rock: > 40°

1.2 Force Balance on a Horizontal Bed

Forces acting on a sediment grain:

1. Drag F_D
2. Lift F_L
3. Submerged weight W'
4. Frictional resistance

$$F_R = \mu(W' - F_L)$$

Threshold condition:

$$F_D = \mu W' - F_L$$

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2. Dimensionless Thresholds and Flow Regimes

2.1 Shields Parameter and Reynolds Number

$$\tau_* = \frac{\tau_0}{(\gamma_s - \gamma)d}$$
$$Re_* = \frac{u_* d}{\nu}$$

Regimes

- **Viscous regime (small Re_*)**: Higher stress required
- **Fully rough regime (large Re_*)**: For $Re_* > 70$, $\tau_* \approx 0.03$–0.06

2.2 Coleman–Ikeda–Iwagaki (C–I–I) Model

Uses particle-level velocity U_p:

$$f(Re_*) = \frac{U_p}{u_*}$$

Regime	Condition	$f(Re_*)$
Hydraulically smooth	$Re_* \lesssim 5$	$Re_*/2$
Transitional	$5 < Re_* < 70$	Interpolated
Hydraulically rough	$Re_* \gtrsim 70$	≈ 6.7

In the viscous limit, τ_{*c} is large because near-bed velocity is small.
In the rough limit, τ_{*c} approaches a constant value.

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3. Threshold Mechanics on Side Slopes

Side slopes fail before beds because gravity contributes an additional driving force.

3.1 Stability Reduction Factor

$$K = \frac{\tau_{*c\alpha}}{\tau_{*c0}}$$

3.2 Analytical Approaches

Approach 1 – Full Mechanical Model (with lift)

- Includes lift normal to bank
- More conservative (lower K)

Approach 2 – Simplified Vector Model (no lift)

$$K = \cos\,\alpha\sqrt{1-\frac{\sin^{2}\alpha}{\sin^{2}\phi}}$$

If $\alpha = \phi$, then $K = 0$.

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4. Stable Channel Design Methodology

4.1 Glover–Florey–Lane (G–F–L) Theory

All grains along the wetted perimeter are simultaneously at threshold.

$$H(y) = H_0\left[\frac{1-\alpha_L\mu\cos\left(\sqrt{\frac{1-\alpha_L\mu}{1+\alpha_L\mu}}\frac{\mu y}{H_0}\right)}{1-\alpha_L\mu}\right]$$

Typical value:

$$\alpha_L = 0.85$$

4.2 Empirical Power-Law Approximations

(ϕ in radians)

$$\frac{B}{H_0} \approx 3.14\phi^{-1.038}$$

$$\frac{A}{BH_0} \approx 0.68\phi^{0.021}$$
$$\frac{P}{H_0} \approx 3.59\phi^{-0.950}$$

4.3 Design Procedures

- **Method A:** Trapezoidal cross-section
- **Method B:** Curved banks with flat base

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5. Comparative Design Example

Given:

- $Q = 5.0 \text{ m}^3/\text{s}$
- $d = 0.010 \text{ m}$
- $\phi = 35°$

Parameter	Method A (Trapezoidal)	Method B (Curved + Flat)
Bank slope (V:H)	1:2.62	Curved
Max depth H_0	0.74 m	0.74 m
Bottom width b	7.08 m	5.98 m
Top width T	11.0 m	9.87 m
Area A	6.69 m²	6.37 m²

Conclusion

Method A is simpler to build.
Method B is more hydraulically efficient and closer to the ideal stable geometry.

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6. Research Limitations

- Threshold is probabilistic
- Shields curve is empirical
- Not valid for cohesive or mixed sediments
- Does not include vegetation or time-dependent scour