Alluvial Bed Forms and Flow Resistance

Mechanisms, Classifications, and Engineering Applications

Executive Summary

Alluvial bed forms—such as ripples, dunes, and antidunes—are morphodynamic structures that strongly influence hydraulic resistance and sediment transport in sand-bed rivers. These bed forms arise from self-organized instabilities produced by sediment–flow interactions.

A critical engineering principle in alluvial channels is that **total bed shear stress** τ_b is not the effective force governing sediment transport. Instead, total resistance must be partitioned into:

$$\tau_b = \tau_b' + \tau_b''$$

where:

- τ_b' = grain shear stress (effective for sediment motion)
- τ_b'' = form drag (caused by pressure differences across bed forms)

Only the grain component τ_b' contributes to sediment transport.

This briefing summarizes:

- Bed-form classification across hydraulic regimes
- The physical instability mechanism governing bed-form evolution (Exner concept)
- Three major resistance-partitioning approaches:
  - Einstein–Barbarossa (depth decomposition)
  - Engelund (slope decomposition)
  - Modern algebraic friction-factor implementation

Accurate regime identification and consistent resistance partitioning are essential for reliable prediction of rating curves and sediment transport rates. In dune-dominated flows, form drag may dominate total resistance, substantially increasing stage for a given discharge.

1. Classification and Characteristics of Bed Forms

Bed forms are classified according to the hydraulic regime in which they develop. The governing parameter is the **Froude number**:

$$Fr = \frac{U}{\sqrt{gh}}$$

which distinguishes gravity-wave-dominated flow from inertia-dominated flow.

Bed Form Classification by Regime

Flow Regime	Bed Form	Phase Relation (Bed vs Surface)	Typical Fr	Primary Resistance
Lower	Ripples	Out of phase	$Fr < 1$	Form roughness
Lower	Dunes	Out of phase	$Fr < 1$	Form drag (separation)
Transition	Washed-out dunes	Variable	$Fr \approx 1$	Variable
Upper	Plane bed	In phase	$Fr \approx 1$	Grain roughness
Upper	Antidunes	In phase	$Fr > 1$	Wave dynamics
Upper	Chutes/Pools	In phase	$Fr > 1$	Continuous transport

Characteristic Profiles

Ripples

- Small-scale: wavelength and height $\ll h$
- Common in hydraulically smooth or transitional flow
- Downstream migration
- Dominated by grain-scale processes

Dunes

- Large-scale: height ≈ $0.1–0.3h$
- Wavelength $L_d \approx 6–7h$
- Asymmetrical shape with slip face
- Strong flow separation and pressure drag

Antidunes

- Occur in supercritical flow
- Strongly influenced by free-surface wave dynamics
- May migrate upstream or downstream

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2. Morphodynamic Instability and the Exner Concept

Bed forms arise from instabilities in the coupled sediment–flow system. Their evolution is governed by the **Exner equation**:

$$\frac{\partial \eta}{\partial t} + \frac{1}{1-\lambda_p}\frac{\partial q_b}{\partial x} = 0$$

where:

- η= bed elevation
- q_b= volumetric sediment transport
- λ_p= bed porosity
- x= streamwise coordinate

Instability Mechanism

1. A small bed perturbation creates a shear-stress perturbation.
2. Shear perturbation causes a transport perturbation.
3. A phase lag between bed topography and transport leads to growth of the bed form.

This feedback loop produces self-organized patterns such as dunes and ripples.

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3. Flow Regime Identification Tools

Correct resistance closure depends on identifying the hydraulic regime.

Key Diagnostic Parameters

Froude Number

$$Fr = \frac{U}{\sqrt{gh}}$$

Distinguishes subcritical (dune) from supercritical (antidune) flow.

Shields Parameter

$$\Theta = \frac{\tau_b}{(\rho_s - \rho)gD}$$

Indicates sediment mobility and transport stage.

Relative Submergence

$$\frac{h}{d}$$

Represents separation between flow depth and roughness scale.

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4. Theory of Resistance Partitioning

In alluvial channels:

$$\tau_b = \tau_b' + \tau_b''$$
$$f = f' + f''$$

Critical Insight

- τ_b'(grain shear) drives sediment transport.
- τ_b''(form drag) results from pressure differences across bed forms.

- Form drag does **not** initiate particle motion.

Using total shear τ_b in sediment-transport formulas overpredicts transport in dune regimes.

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5. Comparison of Resistance Partitioning Methods

Three major approaches are used in engineering practice.

Method Overview

Method	Basis of Decomposition	Empirical Closure	Implementation
Einstein–Barbarossa (1952)	Depth: $R_b = R_b' + R_b''$	Fig. 6.27	Graphical / Iterative
Engelund (1966)	Slope: $S = S' + S''$	Fig. 6.28	Graphical / Iterative
Modern Algebraic	Friction factor: $f = f' + f''$	Roughness predictors	Numerical

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Einstein–Barbarossa (1952)

Core idea: split hydraulic radius, not slope.

- Grain roughness: $k_s' = D_{65}$
- Log-law with transitional correction factor χ
- Form component from empirical U/u_*''–Ψ relation
- Often predicts strong form dominance in dune regime

Engelund (1966)

Core idea: split slope into grain and form components.

- Grain roughness: $k_s' = 2D_{65}$
- Empirical relation between total and grain Shields:

$$\Theta' = 0.06 + 0.4\Theta^2 \text{(lower regime)}$$

Modern Algebraic Friction-Factor Partition

- Uses Darcy–Weisbach framework
- Grain friction based on $k_s' \approx 2\text{–}3D_{90}$
- Total roughness:

$$k_s = k_s' + k_{s,\text{form}}$$

- Dune geometry closure:

$$k_{s,\text{form}} = 11\Delta_d\left(1 - e^{-25\Delta_d/L_d}\right)$$

Commonly implemented in:

- HEC-RAS
- Delft3D
- TELEMAC

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6. Engineering Applications and Observed Trends

Rating Curve Behavior

In sand-bed rivers:

- Dunes increase f'' → higher stage for given discharge.
- As dunes wash out (transition to plane bed), f'' drops rapidly.
- Stage may decrease even as discharge increases.

Comparative Example

(Q = 60 m³/s, S = 0.0004)

Metric	Einstein–Barbarossa	Engelund	Modern Algebraic
Depth h	2.558 m	1.980 m	2.362 m
Grain Shear τ_b'	1.38 Pa	2.46 Pa	1.48 Pa
Form Shear τ_b''	8.65 Pa	5.31 Pa	7.79 Pa
Grain Fraction	0.14	0.32	0.16

Key Insights

Depth Prediction

$$h_{\text{Engelund}} < h_{\text{Modern}} < h_{\text{E\&B}}$$

Grain Fraction Variation
Differences by a factor of two are common. Because:

$$q_b \propto (\tau_b' - \tau_c)^n$$

small differences in τ_b' can lead to large transport discrepancies.

Regime Consistency
All methods correctly identify the lower-regime dune configuration.

7. Essential Modeling Takeaways

1. **Form drag dominates in lower-regime flows.**
2. **Use grain shear for sediment transport calculations.**
3. **Partitioning is less critical in upper-regime plane-bed flows.**
4. **Hydraulics and resistance partition must be iterated consistently.**