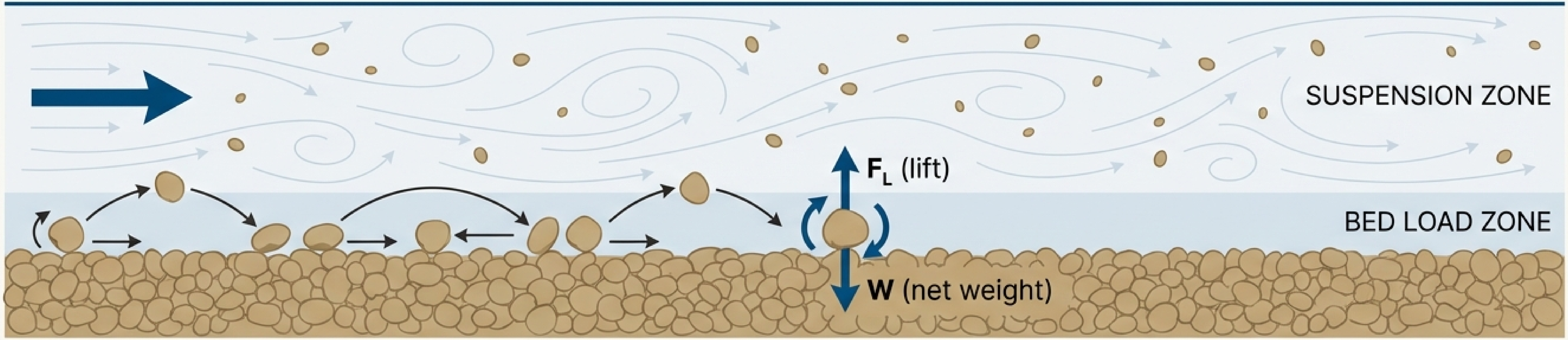
Topic VII: Bed Load Transport

The Physics of Moving Boundaries

From grain-scale mechanics to river morphodynamics.



Bed Load

- Near-bed, **intermittent** contact (rolling, sliding, saltation)
- Gravity-dominated, controlled by near-wall turbulence

Suspended Load

- Entire water column, no bed contact
- Turbulence-supported, continuous suspension

The Transition: The Rouse Number

$$P = \frac{w_s}{\kappa u_*}$$

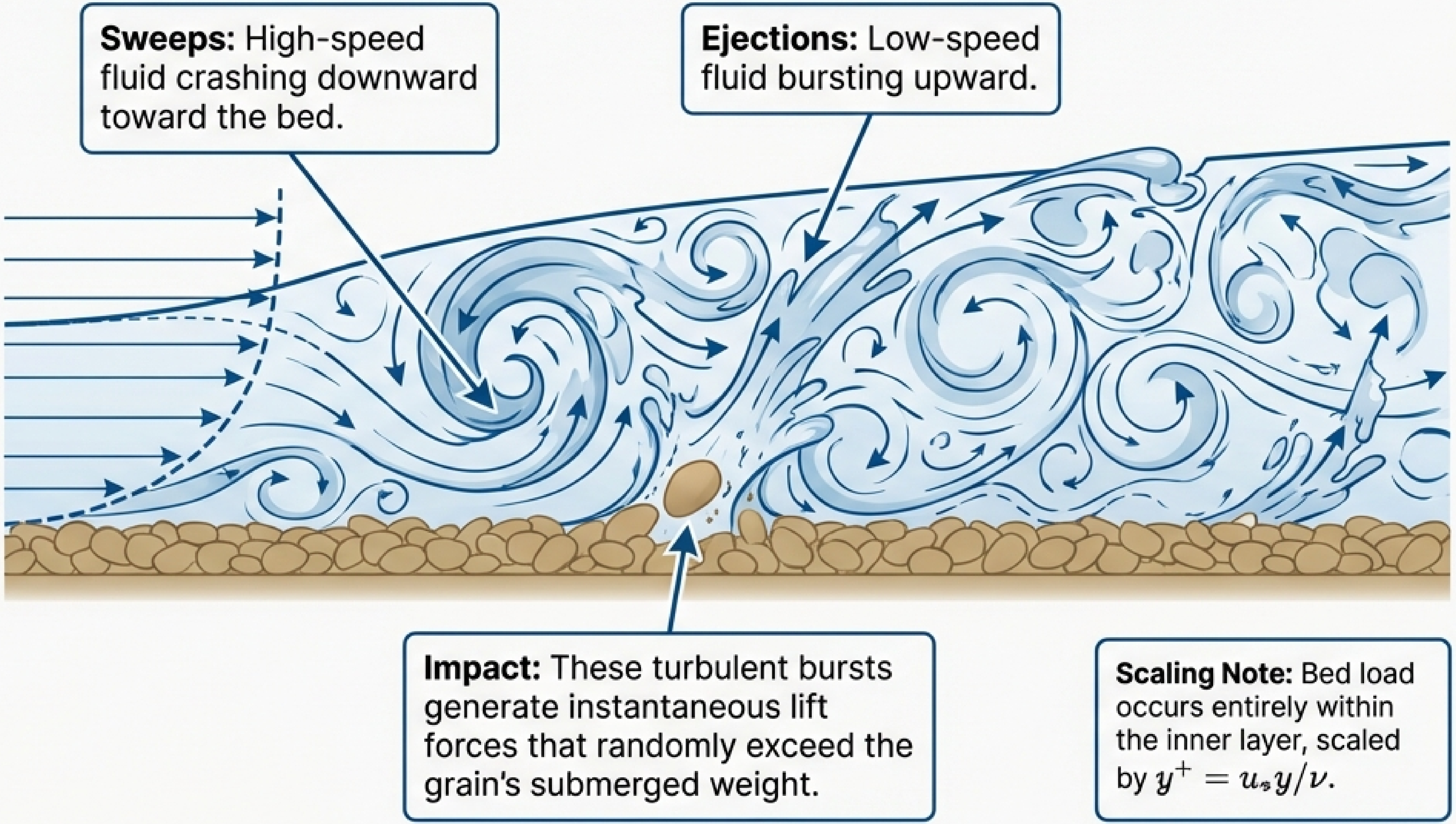
w_s: Particle settling velocity

u_*: Shear velocity (grain-related)

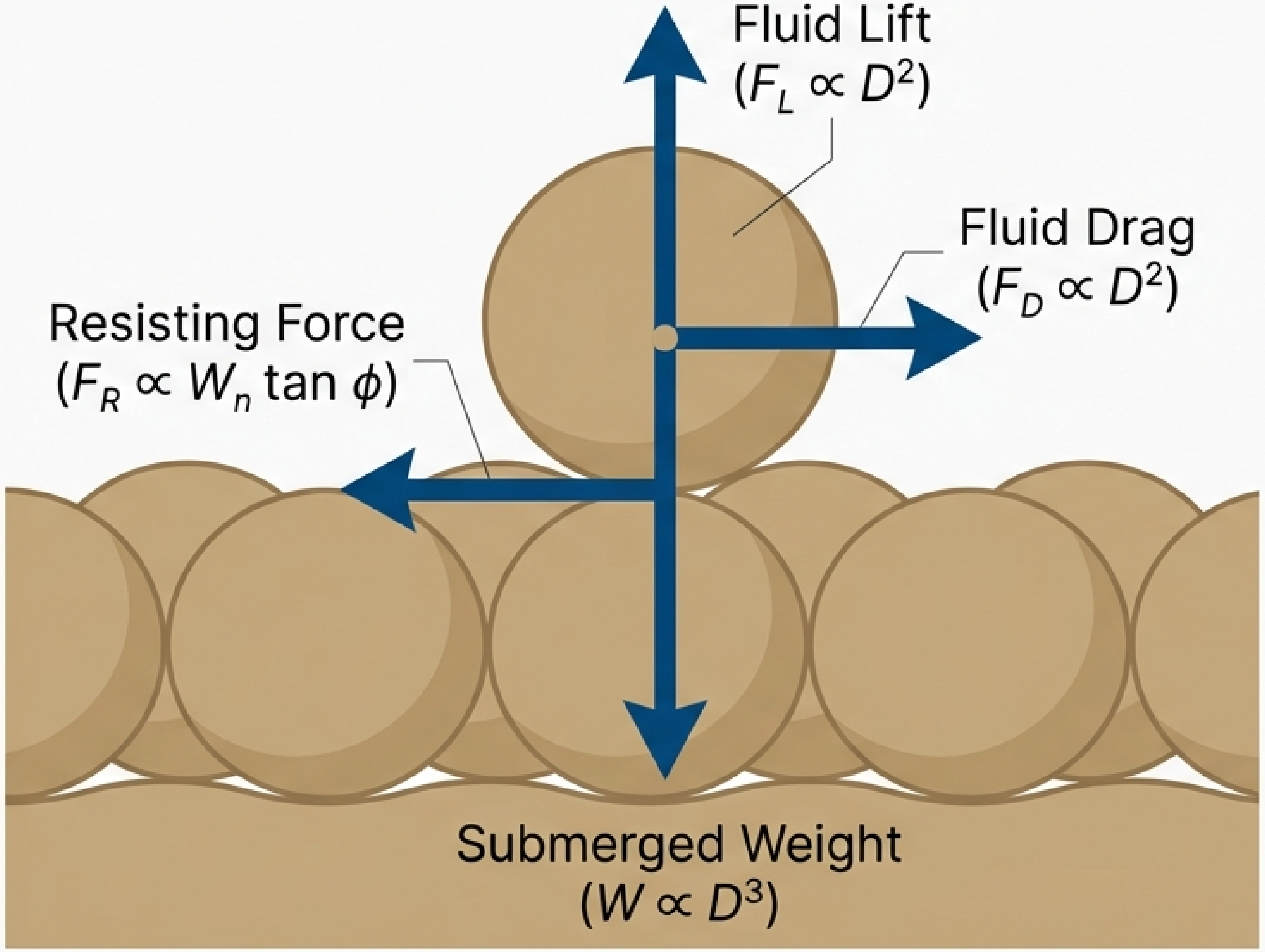
Large P → Bed Load. Small P → Suspension.

The Engine of Entrainment

Average flow velocity ($\overline{u}$) does not determine motion. Bed load is driven by instantaneous turbulent fluctuations (u').



The Grain-Scale Force Balance



The Threshold Equation

$$F_D > (W - F_L)\tan\phi$$

The Scale Mismatch

Because weight scales with the cube of the diameter (D^3) but fluid forces scale with the square (D^2), larger grains require disproportionately larger shear stress to move.

The Shields Parameter (τ_*)

Fluid driving force (Grain shear stress)

$$\tau_* = \frac{\tau'}{(\rho_s - \rho) g D}$$

The universal dimensionless metric for sediment mobility

Submerged resisting weight

Critical Shields Number (τ_{*c}): The statistical threshold of motion.

Excess Shear ($\tau_* - \tau_{*c}$): The energy available to actually transport sediment.

All modern transport formulas collapse into the unified form: $\Phi_b = f(\tau_*)$.

CRITICAL CONCEPT: The Shear Partition

$$\tau = \tau' + \tau''$$

τ (Total Shear): The total energy gradient of the river.

τ' (Grain Shear): Skin friction. **This is what actually moves grains.**

τ'' (Form Drag): Energy lost to turbulence over dunes and ripples.

All bed load formulas require τ' (Grain Shear). Form drag from dunes does NOT move sediment. Using Total Shear (τ) when bedforms are present will catastrophically overpredict transport.

The Deterministic Approach (Excess Shear)

Transport rate is simply the volume of moving sediment multiplied by how fast it is moving.

$$q_b = N_a \times V \times U_s$$

- N_a: Number of moving grains per unit area.
- V: Volume per grain $\left(\frac{\pi}{6} D^3\right)$.
- U_s: Mean grain velocity.

Key Insight: Derived from force balances, grain velocity saturates at high shear but fundamentally scales with excess shear velocity:

$$U_s \approx 8.5(U_* - 0.9U_{*c})$$

The Meyer-Peter & Müller (MPM) Formula

$$\Phi_b = 8(\tau_* - \tau_{*c})^{3/2}$$

Why the 3/2 Power?

- **Factor 1:** Number of moving grains $\propto (\tau_* - \tau_{*c})$
- **Factor 2:** Grain velocity $\propto (\tau_* - \tau_{*c})^{1/2}$
- **Result:** Multiplying them yields the 3/2 exponent. It is an empirical formula, but completely physically interpretable.

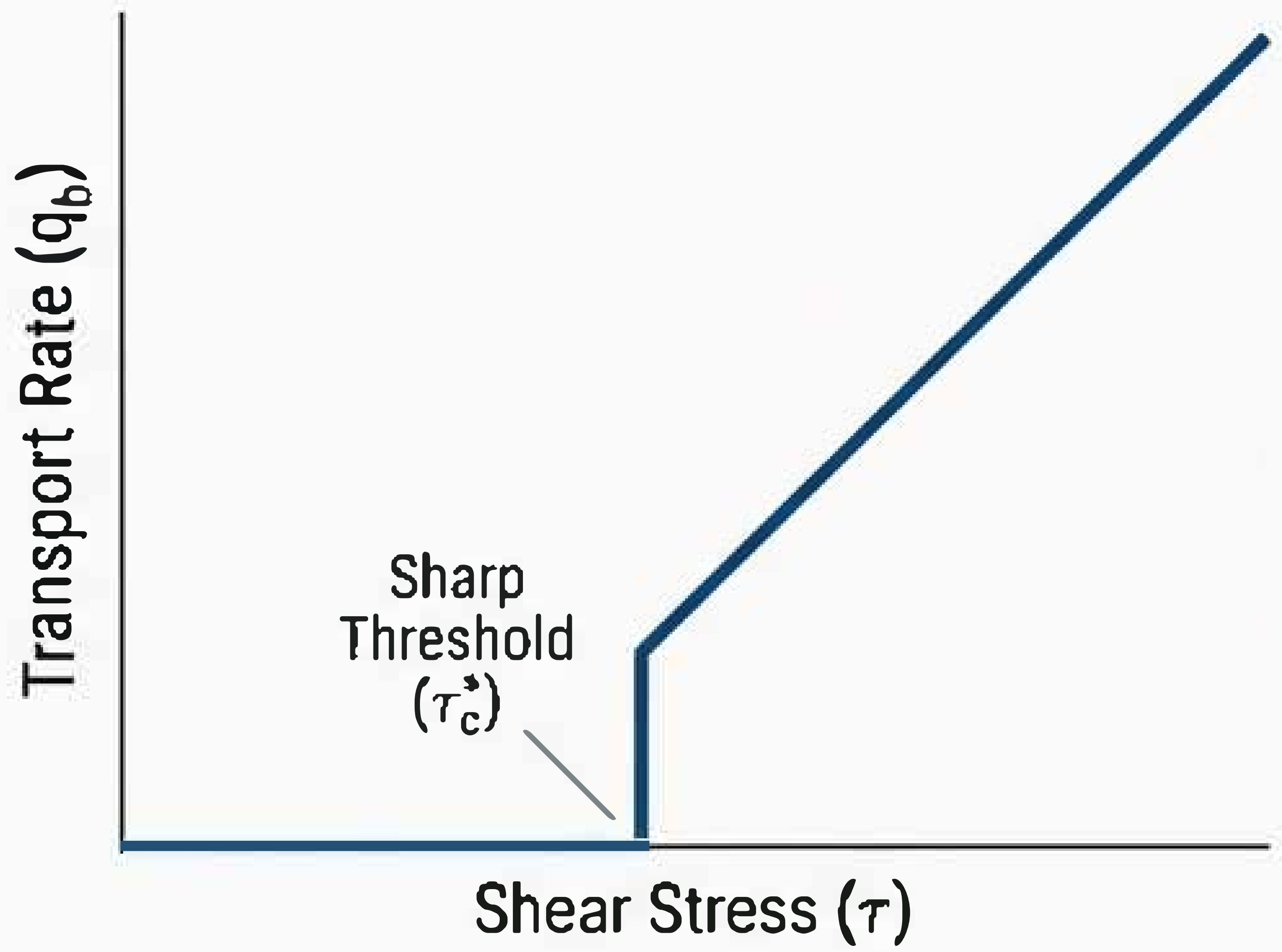
Sidebar Note

Fernandez Luque & van Beek:

$$\Phi_b = 5.7(\tau_* - \tau_{*c})^{3/2}$$

A refined calibration that proves the robustness of the 3/2 structural form.

The Limit of Excess Shear



The Einstein Revolution (1942)

$$P = \text{Prob}\left(\frac{F_L}{W} > 1\right)$$

$$\frac{F_L}{W} > 1 \Rightarrow \frac{F_L(1+\eta)}{W} > 1$$

$$\Rightarrow 0.135\frac{\kappa^2 U_*^2}{\rho g d}(1+\eta) > 1$$

where $\kappa = \frac{U_f}{U_*}$

$$\Rightarrow (1+\eta) > \frac{7.41\Psi^*}{\kappa^2}$$

where $\Psi^* = \frac{1}{\tau_*} = \frac{Rgd}{{U_*}^2} = \frac{\rho Rgd}{\tau_0}$

- Assumes a sharp, rigid threshold for motion.
- Ignores turbulence intermittency.
- Inaccurate near the threshold where transport actually begins.

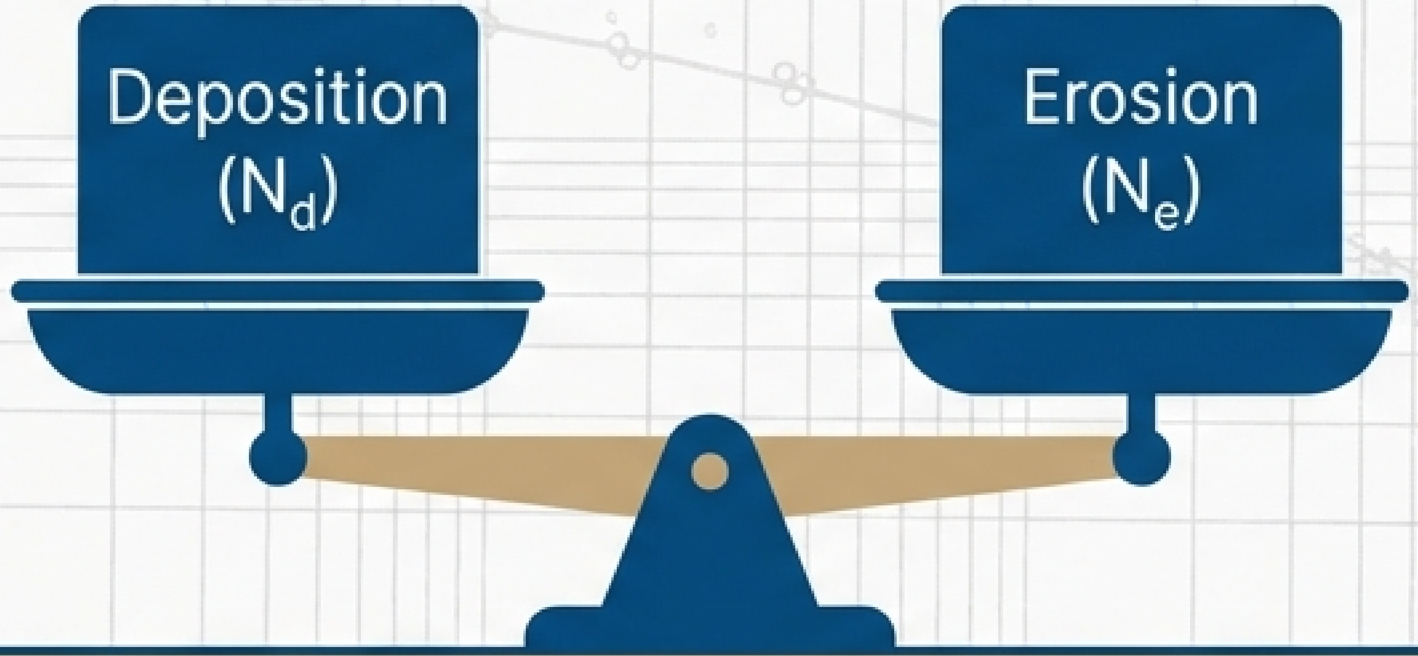
The Probabilistic Shift

Entrainment is modeled statistically:

$P = \text{Prob}(F_L/W > 1)$.

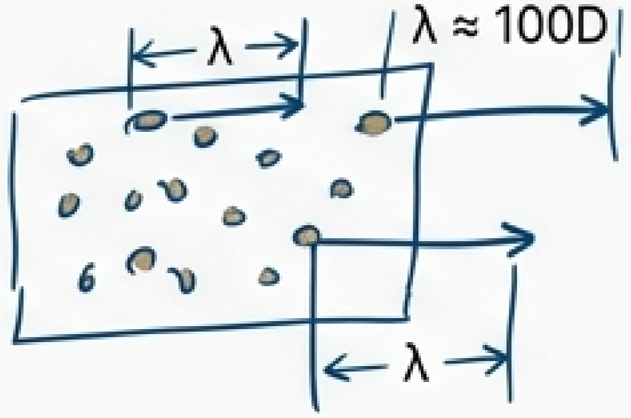
- Turbulent lift fluctuates randomly (Gaussian distribution).
- Motion is highly intermittent.
- There is no sharp threshold.

Einstein's Equilibrium Framework



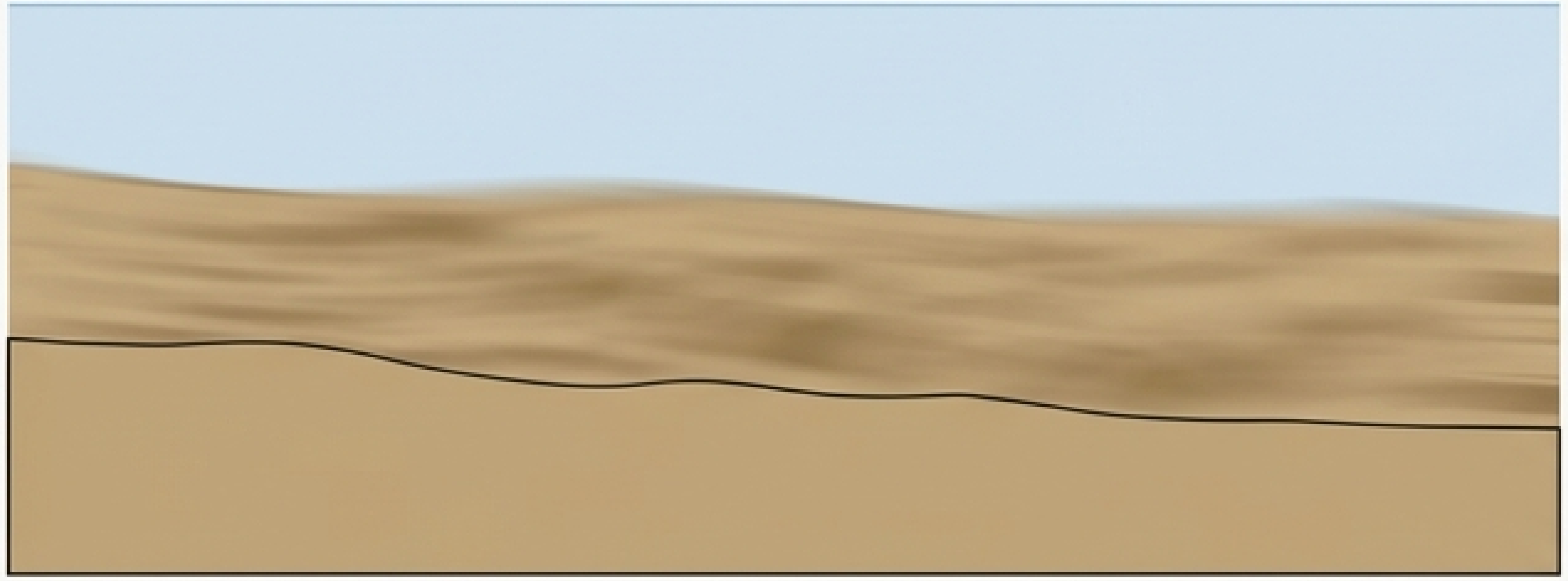
The 3 Fundamental Assumptions

1. **Motion** occurs when instantaneous lift > weight.
2. The **Mean Jump Length**: A particle jumps a consistent distance of $\lambda \approx 100D$ between deposition points.
3. **Equilibrium**: The statistical rate of Deposition (N_d) equals Erosion (N_e).



Takeaway: This yields an implicit function showing transport increasing gradually, not abruptly.

Extreme Scales: Sheet Flow Regime



- Triggered when $\tau^* > 0.8 - 1.0$.
- Discrete saltation vanishes. The bed itself becomes a thick, mobile granular fluid layer.
- Critical for coastal storms and tsunamis.

Macro-Scale: The Exner Equation

A divergence in bed load transport

$$(1 - \lambda_p)\frac{\partial \eta}{\partial t} + \frac{\partial q_b}{\partial x} = 0$$

directly causes the riverbed to erode or deposit

Takeaway: Bed load transport is the engine that drives river morphodynamics.

Theory to Practice: Numerical Example

A wide alluvial river (B=50m, H=3.0m, S=0.0003, D_{50}=0.60mm). Assume dunes exist.

1. Hydraulics

Calculate Total Bed Shear ($\tau = \rho g H S =$ = 8.829 Pa).

2. Total Resistance

Compute Total Friction Factor ($f = 0.03973$).

3. Skin Friction

Compute Grain Friction Factor ($f' = 0.01303$) using $k_s \approx 2.5 D_{50}$.

4. THE PARTITION

Extract Grain Shear:

$\tau' = \tau(f'/f) =$ = 2.896 Pa.

(Note: τ' is only ~33% of total τ!)

5. Calculate

Compute τ_* (0.298) and apply MPM formula to get $q_b = 5.95 \times 10^{-5}\ \mathrm{m^2/s}$.

The Cost of Ignoring the Partition

Partition Method	Total Shear τ (Pa)	Grain Shear τ' (Pa)	MPM Result (q_b)
Modern Algebraic Friction	8.829	2.896	5.95×10^{-5}
Einstein-Barbarossa	8.829	3.42	7.97×10^{-5}
Engelund	8.829	2.43	4.33×10^{-5}

Core Takeaways

- Because bed load scales to the 3/2 power, tiny errors in τ' create massive engineering errors in predicted transport.
- Near-wall turbulence drives entrainment.
- MPM offers practical determinism; Einstein provides probabilistic realism.
- Always use Grain Shear, never Total Shear.