Physical Foundations and Engineering Models of Bed Load Transport

Briefing Document

Executive Summary

Bed load transport is a fundamental process in river morphodynamics, coastal engineering, and channel design. Unlike suspended load, bed load refers to sediment that moves in intermittent contact with the channel bed through rolling, sliding, or saltation.

This document synthesizes the physical mechanics of grain motion, the role of turbulent boundary layers, and the two primary modeling frameworks used to predict transport rates: deterministic (excess shear) models and Einstein's probabilistic theory.

Critical Takeaways

- **Turbulence-Driven Entrainment**
  Grain motion is governed not by mean flow velocity, but by instantaneous turbulent fluctuations (e.g., sweeps and ejections) within the near-wall region.
- **Shear Stress Partitioning**
  Only the grain shear stress (τ')contributes to sediment transport. Total shear stress (τ)includes form drag (τ'')from bedforms (e.g., dunes), which does not mobilize grains. Using total shear leads to overestimation.
- **Modeling Dichotomy**
  The Meyer–Peter & Müller (MPM) model provides a practical deterministic framework based on excess shear, whereas Einstein's probabilistic theory captures the stochastic nature of turbulence and is more rigorous near the threshold of motion.
- **Morphodynamic Coupling**
  Bed load transport is directly linked to channel evolution through the Exner equation, where spatial changes in transport control erosion and deposition.

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1. Physical Foundations of Bed Load

1.1 Definition and Identification

Bed load refers to sediment transported within a thin near-bed layer, characterized by:

- **Intermittent Contact**: Motion occurs in bursts rather than continuously

- **Gravity Dominance**: Gravity resists motion, unlike suspended load where turbulence supports particles
- **Modes of Transport**: Rolling, sliding, and saltation (short jumps)

1.2 Bed Load vs. Suspended Load

The transition between transport modes is governed by the **Rouse number**:

$$P = \frac{w_s}{\kappa u_*}$$

where:

- w_s: settling velocity
- $\kappa \approx 0.40$: von Kármán constant
- u_*: shear velocity (grain-related)

Feature	Bed Load	Suspended Load
Zone of Motion	Near-bed layer	Entire water column
Contact	Intermittent / maintained	No bed contact
Supporting Force	Gravity-dominated	Turbulence-supported
Motion Type	Rolling, sliding, saltation	Continuous diffusion

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2. Turbulence and Boundary Layer Physics

Bed load transport is controlled by the **inner region of the turbulent boundary layer**.

2.1 Turbulent Structure

Instantaneous velocity is decomposed as:

$$u = \bar{u} + u'$$

Near the bed, key turbulent events drive entrainment:

- **Sweeps**: High-speed fluid moving downward
- **Ejections**: Low-speed fluid moving upward
- **Bursting Events**: Generate instantaneous lift forces that can exceed submerged grain weight

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2.2 Wall Scaling

The relevant scaling parameter is:

$$y^{+} = \frac{u_{*}y}{\nu}$$

Turbulence intensity peaks near $y^{+} \approx 100$. Because entrainment depends on near-wall turbulence, transport scales with **shear velocity** u_{*}rather than bulk velocity.

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3. Grain-Scale Mechanics and Thresholds

3.1 Force Balance

Grain motion begins when fluid forces exceed resisting forces:

$$F_D > (W - F_L)\tan\ \phi$$

- Drag and lift: $\propto D^2$
- Submerged weight: $\propto D^3$

Thus, larger grains require disproportionately greater shear stress.

3.2 Shields Parameter

$$\tau_{*} = \frac{\tau'}{(\rho_s - \rho)gD}$$

- τ_{*c}: critical Shields parameter
- $\tau_{*} - \tau_{*c}$: excess shear driving transport

3.3 Shear Stress Partitioning

$$\tau = \tau' + \tau''$$

- τ': grain shear (drives transport)
- τ'': form drag (energy loss to bedforms)

Key rule: All transport formulas must use τ', not τ.

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4. Deterministic Excess Shear Models

These models assume a **sharp threshold** for motion.

4.1 Dimensionless Bed Load Function

$$\Phi_b = \frac{q_b}{\sqrt{(s-1)gD^3}}, \Phi_b = f(\tau_*)$$

4.2 Meyer–Peter & Müller (MPM, 1948)

$$\Phi_b = 8(\tau_* - \tau_{*c})^{3/2}$$

Origin of the $3/2$power:

- Number of moving grains ∝ $(\tau_* - \tau_{*c})$
- Grain velocity ∝ $(\tau_* - \tau_{*c})^{1/2}$

$$\Rightarrow (\tau_* - \tau_{*c})^{3/2}$$

4.3 Alternative Models

- **Fernandez Luque & van Beek**:

$$\Phi_b = 5.7(\tau_* - \tau_{*c})^{3/2}$$

- **Limitation**:
  These models ignore turbulence intermittency and perform poorly near incipient motion.

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5. Einstein’s Probabilistic Theory

Einstein (1942/1950) proposed that entrainment is **stochastic**, not deterministic.

5.1 Fundamental Assumptions

1. **Stochastic entrainment**: Motion occurs when instantaneous lift exceeds weight
2. **Mean jump length**: $\lambda \approx 100D$
3. **Statistical equilibrium**: $N_e = N_d$

5.2 Probability-Based Transport

Assuming Gaussian turbulence:

- Transport increases **gradually**, not abruptly
- No strict threshold exists

Rates are defined as:

- **Erosion**:

$$N_e = \frac{P}{(\pi/4)D^2 t_e}$$

- **Deposition**:

$$N_d = \frac{q_b}{[(\pi/6)D^3]\lambda D}$$

The resulting relationship is expressed graphically using:

$$\Psi_* = \frac{1}{\tau_*}, \Phi_*$$

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6. Advanced Regimes and Morphodynamics

6.1 Sheet Flow Regime

For very high shear:

$$\tau_* > 0.8\text{–}1.0$$

- Bed becomes a dense, mobile granular layer
- No discrete saltation
- Important in coastal storms and tsunami conditions

6.2 Surface-Based Models (Mixed Sediment)

- **Hiding effect**: small grains shielded
- **Exposure effect**: large grains more exposed

Models:

- Parker (1979)
- Wilcock & Crowe (2003)

6.3 Exner Equation

$$(1-\lambda_p)\frac{\partial \eta}{\partial t}+\frac{\partial q_b}{\partial x}=0$$

- $\partial q_b / \partial x > 0$: erosion
- $\partial q_b / \partial x < 0$: deposition

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7. Comparative Summary of Approaches

Feature	Deterministic (Excess Shear)	Einstein (Probabilistic)
Complexity	Simple, empirical	Physically rigorous
Threshold	Sharp	Gradual
Use	Engineering design	Theoretical analysis
Weakness	Poor near threshold	Complex implementation
Input	Excess grain shear	Turbulent lift statistics

Implementation Workflow

To compute bed load in practice:

1. **Hydraulics**
   Determine depth, slope, and velocity
2. **Total Shear**

$$\tau = \rho g H S$$

3. **Partitioning**
   Determine grain shear τ'
4. **Shields Parameter**
   Compute τ_*
5. **Transport Model**
   Apply MPM or Einstein → compute Φ_b→ convert to q_b