Physical Foundations and Modeling of Bed-Load Transport

Executive Summary

Bed-load transport is a fundamental process in river morphodynamics, channel design, and coastal engineering. It involves sediment moving in intermittent contact with the bed through rolling, sliding, or saltation. Unlike suspended load, bed load is gravity-dominated and confined to a thin near-bed layer.

A central physical insight is that sediment entrainment is driven not by mean flow velocity, but by **near-wall turbulence**—specifically sweeps, ejections, and instantaneous lift fluctuations.

Two principal modeling frameworks dominate bed-load prediction:

1. **Deterministic Excess Shear Models**
   (e.g., Meyer–Peter & Müller), which assume a threshold of motion and relate transport rate to the 3/2 power of excess shear stress.
2. **Einstein's Probabilistic Theory**
   which treats entrainment as a stochastic process governed by turbulent fluctuations and rejects the idea of a sharp threshold.

A critical requirement in all calculations is **shear stress partitioning**. Only the grain-related (skin) shear stress τ'contributes to sediment motion; form drag $\tau''$is dissipated in bedform resistance. Using total shear instead of grain shear leads to substantial overprediction of transport.

Finally, bed load governs channel evolution through the **Exner equation**, where the spatial divergence of transport determines erosion or deposition.

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1. Characterizing Bed-Load Motion

Bed load consists of sediment transported within a thin near-bed layer while maintaining intermittent contact with the bed.

Key Features

- **Modes of motion:** rolling, sliding, saltation
- **Contact:** intermittent bursts of movement
- **Dominant force:** gravity
- **Driving mechanism:** turbulent fluctuations near the bed

1.1 Bed Load vs. Suspended Load

The transition between bed load and suspension is governed by the **Rouse number**:

$$P = \frac{w_s}{\kappa u_*}$$

Where:

- w_s= settling velocity
- $\kappa \approx 0.40$= von Kármán constant
- u_*= grain shear velocity

Feature	Bed Load	Suspended Load
Location	Near bed	Entire water column
Motion	Intermittent	Continuous
Support	Gravity-dominated	Turbulence-supported
Bed contact	Maintained	None
Condition	Large *P*	Small *P*

2. Turbulent Boundary Layer Physics

Incipient motion is probabilistic because near-bed flow is dominated by turbulence.

Instantaneous velocity:

$$u = \bar{u} + u'$$

Transport is driven by fluctuations u', not by mean velocity.

2.1 Near-Bed Turbulence Structures

- **Sweeps:** high-speed fluid moving downward
- **Ejections:** low-speed fluid moving upward
- **Bursting events:** generate instantaneous lift exceeding submerged weight

Thus, particles can move even when mean forces suggest stability.

2.2 Wall Scaling

The relevant scaling parameter is:

$$y^+ = \frac{u_* y}{\nu}$$

Bed load occurs in the inner layer, where turbulence intensity peaks (around $y^+ \approx 100$).

Transport therefore scales with **shear velocity** u_*, not bulk velocity.

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3. Grain-Scale Mechanics and Dimensional Analysis

3.1 Force Balance

Motion begins when:

$$F_D > (W - F_L)\tan\,\phi$$

Scaling:

- Submerged weight $W \sim D^3$
- Drag and lift $F_D, F_L \sim D^2$

Thus, larger grains require disproportionately higher shear stress.

3.2 The Shields Parameter

$$\tau_* = \frac{\tau'}{(\rho_s - \rho)gD}$$

- Represents fluid force relative to submerged weight
- τ_{*c}: statistical threshold of motion
- Excess shear $(\tau_* - \tau_{*c})$drives deterministic models

3.3 Shear Stress Partitioning

In alluvial channels:

$$\tau = \tau' + \tau''$$

- τ': grain (skin) shear → drives transport
- τ'': form drag → lost to bedforms

Rule: All bed-load formulas must use τ'.
Using total shear overestimates transport.

4. Deterministic (Excess Shear) Formulations

Bed-load transport is modeled as:

$$q_b \sim N_a V U_s$$

Where:

- N_a= number of moving grains
- V= grain volume
- U_s= grain velocity

4.1 Meyer–Peter & Müller (1948)

$$\Phi_b = 8(\tau_* - \tau_{*c})^{3/2}$$

- Widely used engineering formula
- 3/2 power arises from:
  - Number of grains ∝ excess shear
  - Velocity ∝ square root of excess shear

4.2 Other Deterministic Models

- Fernandez Luque & van Beek:

$$\Phi_b = 5.7(\tau_* - \tau_{*c})^{3/2}$$

Limitations:

- Assume a sharp threshold
- Do not explicitly represent turbulence intermittency
- Less accurate near incipient motion

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5. Einstein's Probabilistic Theory

Einstein (1942, 1950) treated entrainment as stochastic.

5.1 Core Assumptions

1. Entrainment occurs when instantaneous lift exceeds weight
2. Average jump length ≈ 100D
3. Erosion equals deposition in equilibrium

5.2 Key Insights

- No absolute threshold
- Transport increases gradually

- Probability modeled using Gaussian turbulence

Einstein's framework better captures near-threshold behavior.

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6. High-Shear and Mixed-Sediment Regimes

6.1 Sheet Flow

When:

$$\tau_* > 0.8\text{–}1.0$$

The bed becomes a dense granular layer.

Relevant in:

- Coastal storms
- Tsunami conditions

6.2 Surface-Based Models

For mixed sediment:

- Parker (1979)
- Wilcock & Crowe (2003)

These include:

- Surface grain size effects
- Hiding and exposure mechanisms

6.3 Morphodynamics — The Exner Equation

$$(1-\lambda_p)\frac{\partial \eta}{\partial t}+\frac{\partial q_b}{\partial x}=0$$

Where:

- λ_p= porosity
- η= bed elevation

Erosion and deposition are governed by spatial changes in bed-load transport.

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7. Comparative Summary

Approach	Basis	Threshold	Best Use
Deterministic (MPM)	Empirical	Sharp	Practical engineering
Probabilistic (Einstein)	Statistical	No absolute threshold	Near-threshold theory
Surface-based	Grain interaction	Size-dependent	Gravel rivers
Sheet flow	Granular physics	Very high stress	Coastal extremes

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Final Engineering Takeaway

Accurate bed-load prediction requires:

1. Determining hydraulics
2. Partitioning total shear to obtain grain shear τ'
3. Computing the Shields parameter
4. Applying an appropriate transport function

Because transport scales as:

$$q_b \propto (\tau_* - \tau_{*c})^{3/2}$$

small errors in resistance partitioning produce large errors in predicted transport.