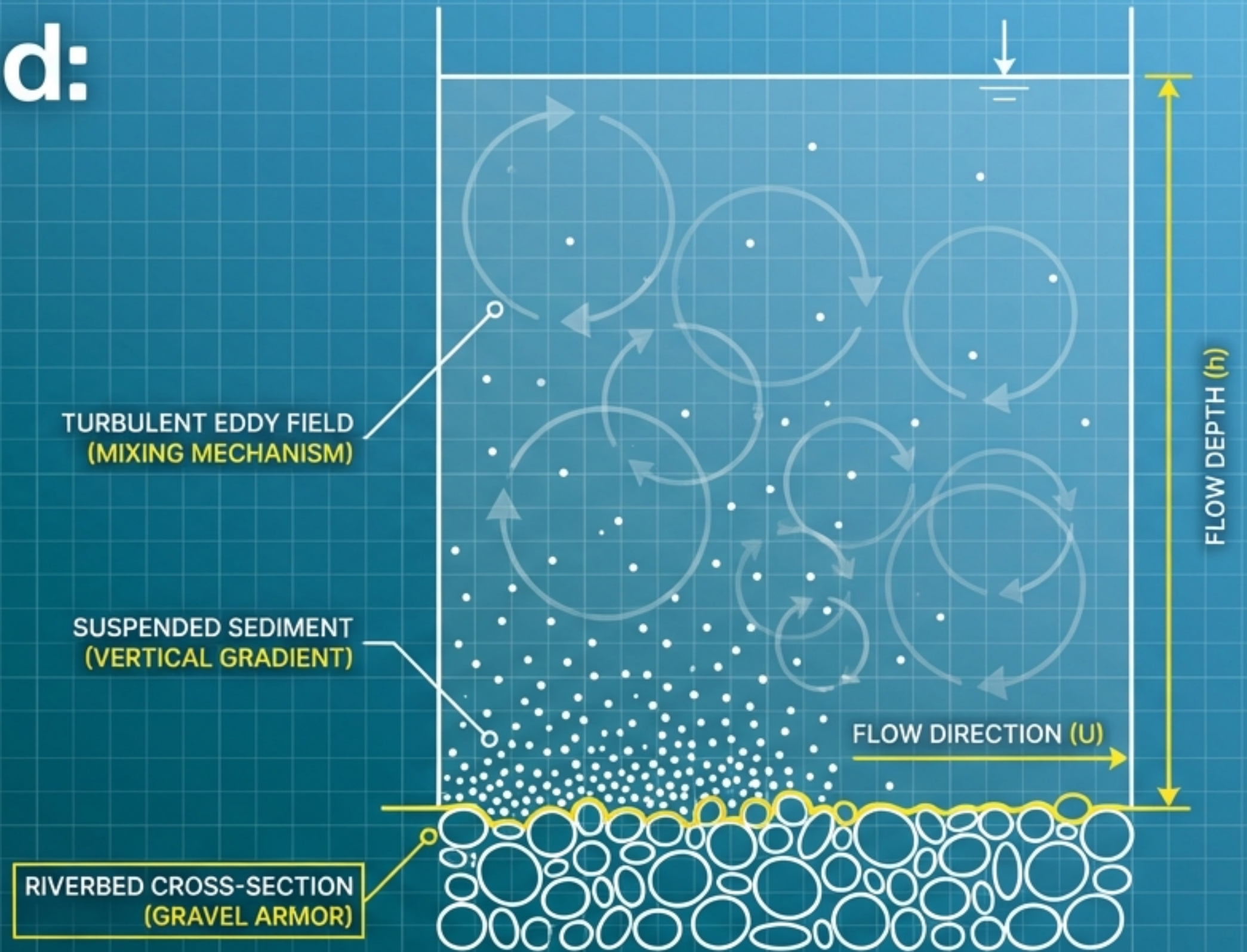


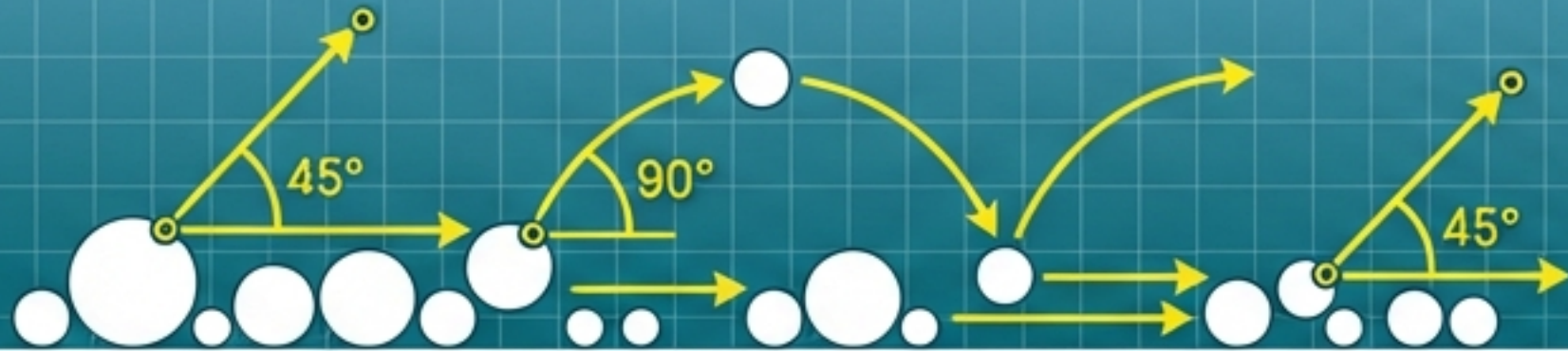
Suspended Load: Mechanics, Modeling, and Computation

A complete physical and mathematical framework for sediment transported by turbulent fluid mixing.



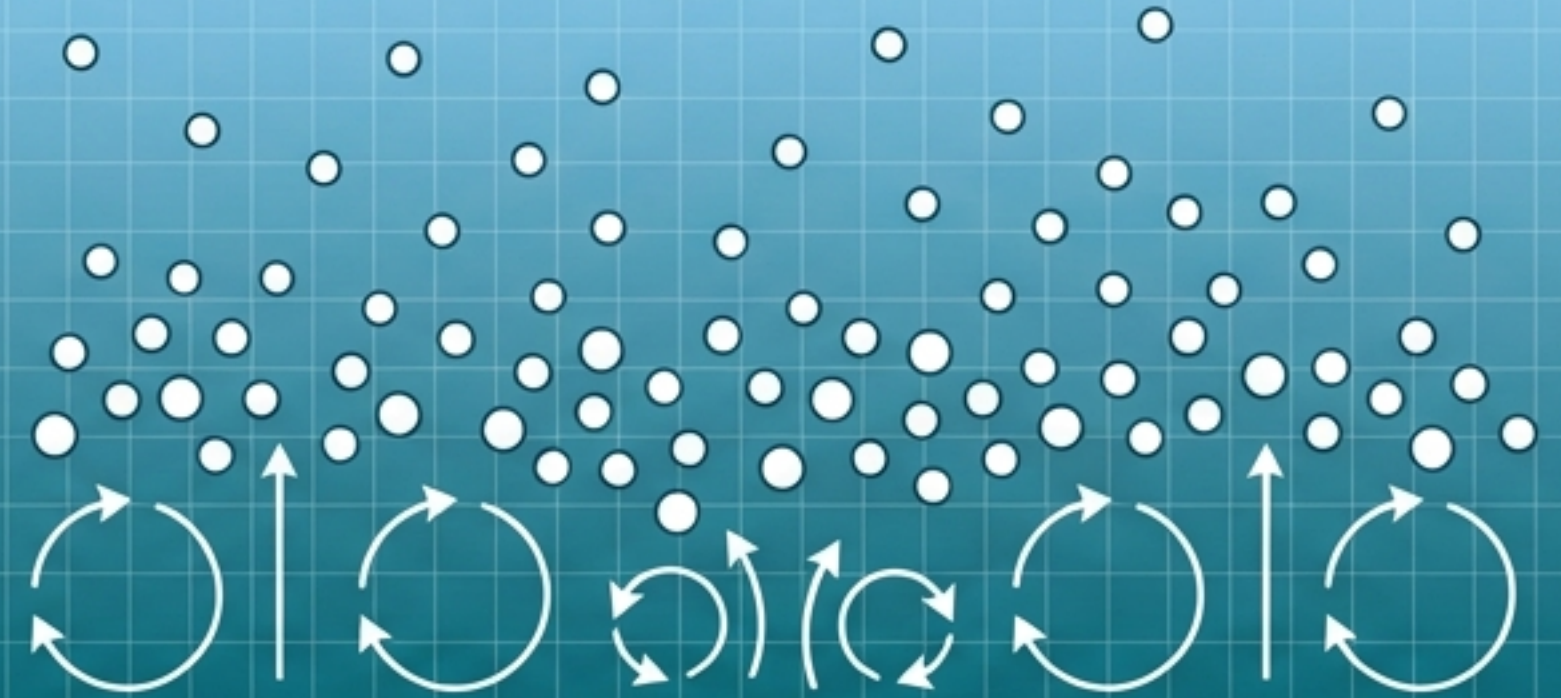
The Two Modes of Transport

Bed Load



- Particles remain in continuous or intermittent contact with the bed.
- Motion is driven directly by near-bed shear stress.
- Moves by rolling, sliding, and saltation.

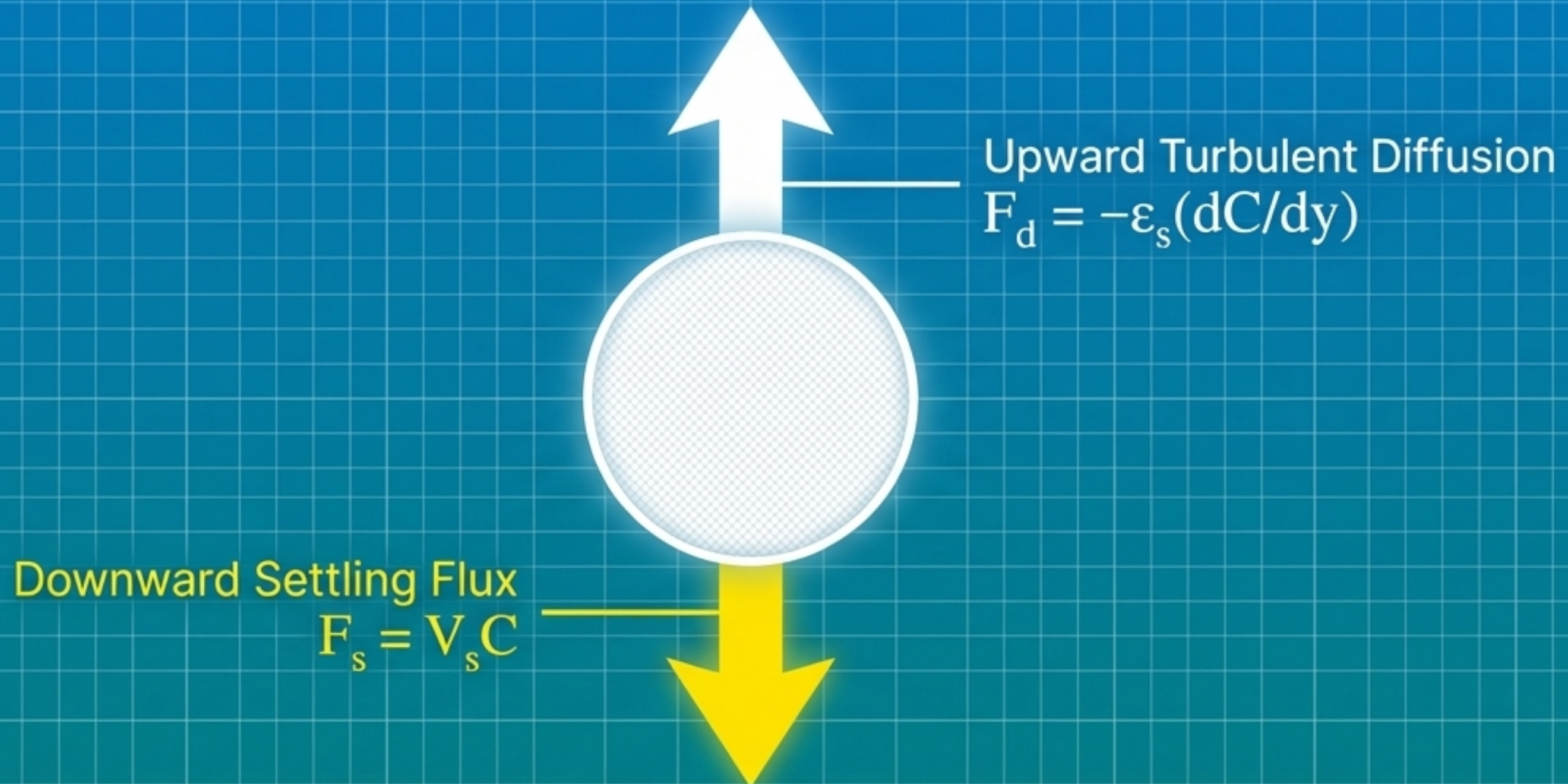
Suspended Load



- Particles are supported entirely by upward turbulent fluid mixing.
- Motion occurs vertically and longitudinally throughout the water column.
- Governed by the balance of gravity and turbulence.

Note: In natural rivers, these two modes operate as a continuous system, transitioning near the bed.

The Mechanics of Suspension



Suspension only occurs when these two competing fluxes achieve perfect balance, yielding a stable vertical concentration.

The Engine of Entrainment: Coherent Structures

While mean vertical velocity is zero ($v = 0, w = 0$), instantaneous turbulent fluctuations (u', v', w') are the sole mechanism lifting sediment.

1. Sweeps

High-speed fluid moves downward toward the boundary



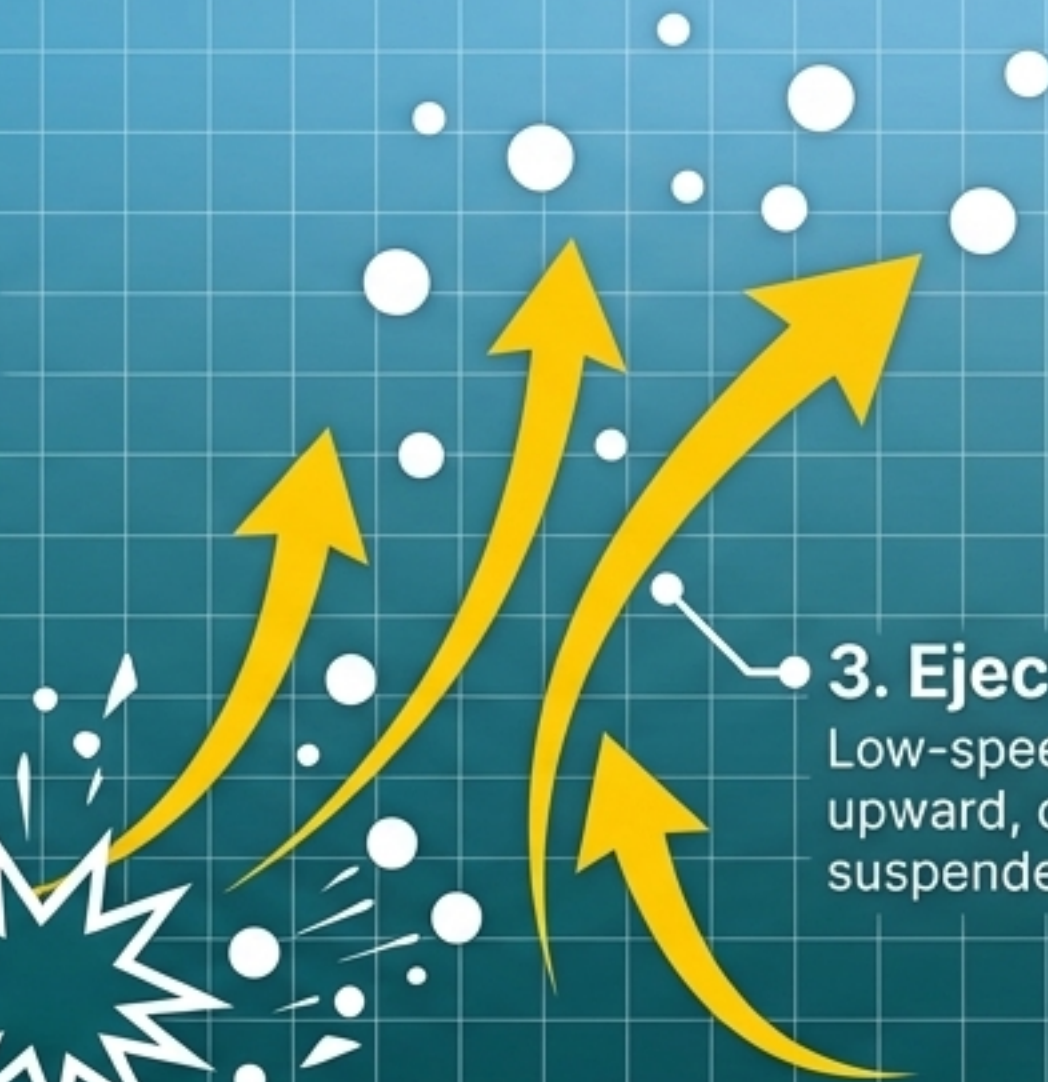
2. Bursting Events

Intense local shear forces break particles free



3. Ejections

Low-speed fluid erupts upward, carrying suspended mass.



Governing the Water Column

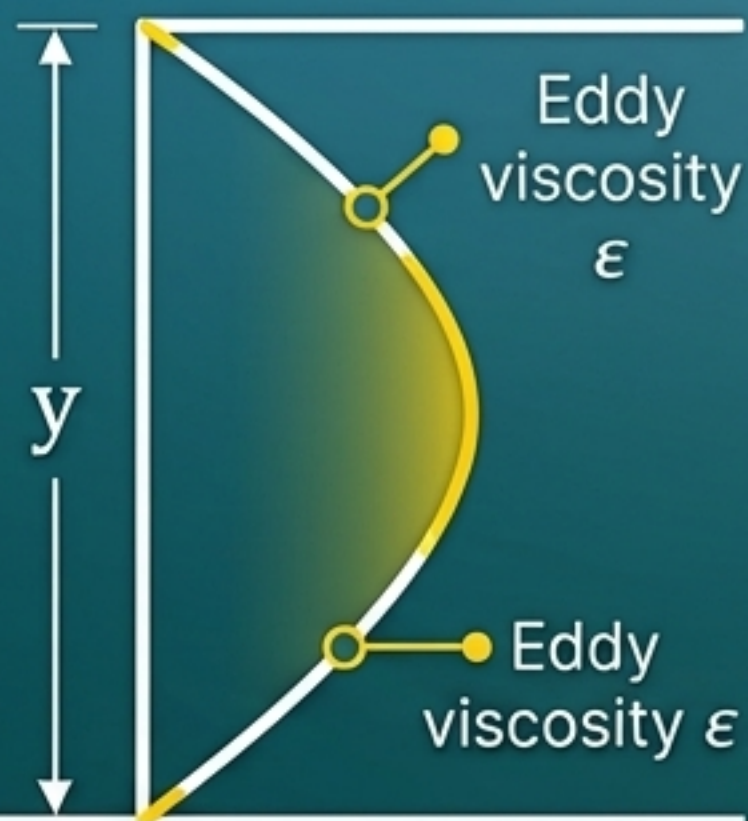
$$V_s C + \varepsilon_s (dC/dy) = 0$$

Sediment Diffusivity (ε_s)

Sediment does not diffuse exactly like momentum due to particle inertia.

$$\varepsilon_s = \beta_s \varepsilon$$

(where $0.5 < \beta_s < 2$)



Velocity Distribution (u)

The classical logarithmic velocity profile defines the advective transport.

$$u = \frac{u_*}{\kappa} \ln \left(\frac{30y}{k_s} \right)$$

Crucial distinction: When bed forms exist, total shear velocity must be replaced by grain shear velocity (u_*') to isolate forces acting on the particles.

The Rouse Concentration Distribution

Relative Concentration

The concentration at elevation y , scaled against the known reference.
Inter

$$\frac{C}{C_a} = \left[\frac{H - y}{y} * \frac{a}{H - a} \right]^z$$

Flow Boundary

Total depth of the water column.
Inter

Anchor Elevation

Helvetica Now Display
The reference height near the bed where sediment transitions to suspension.
Inter

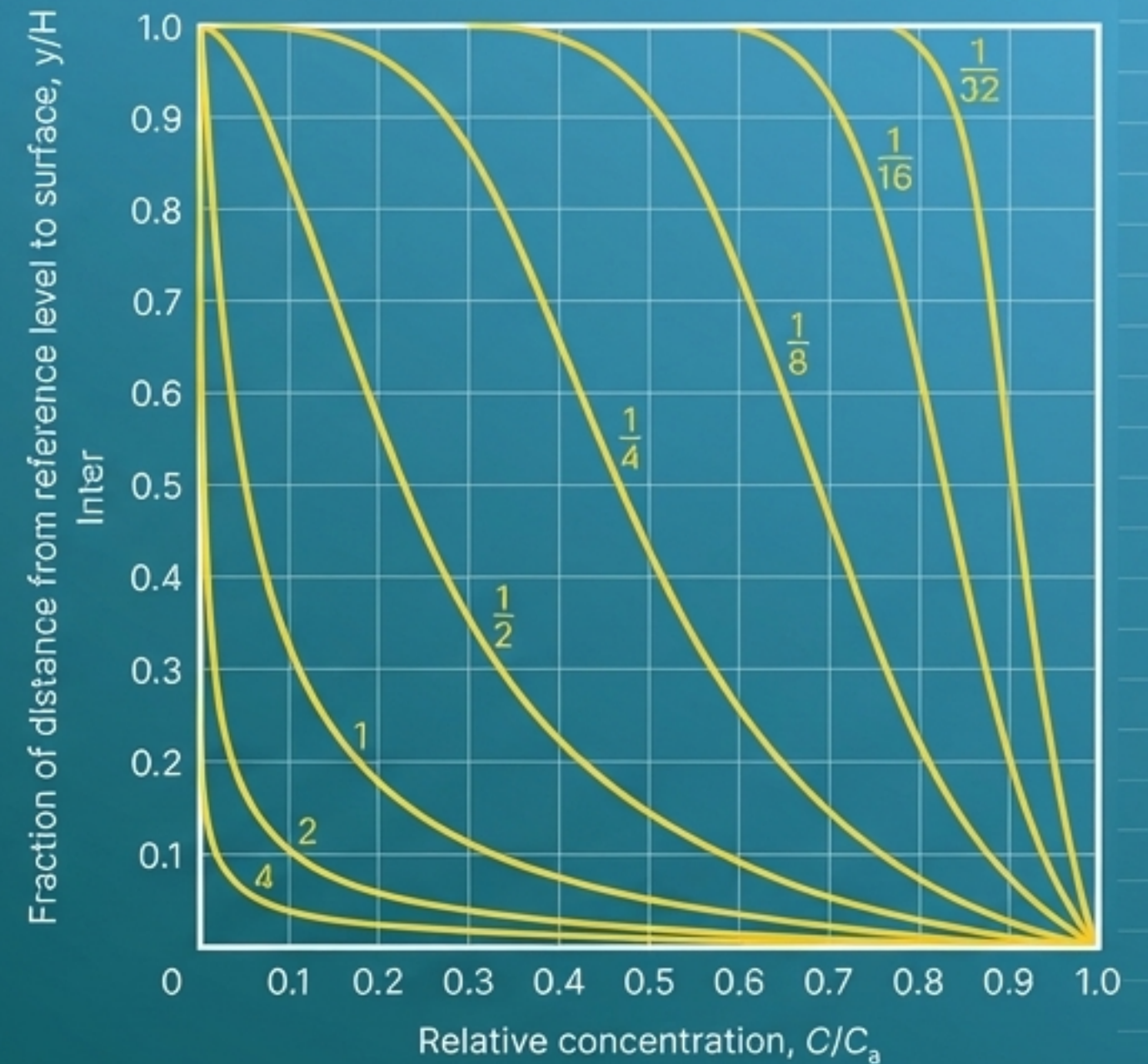
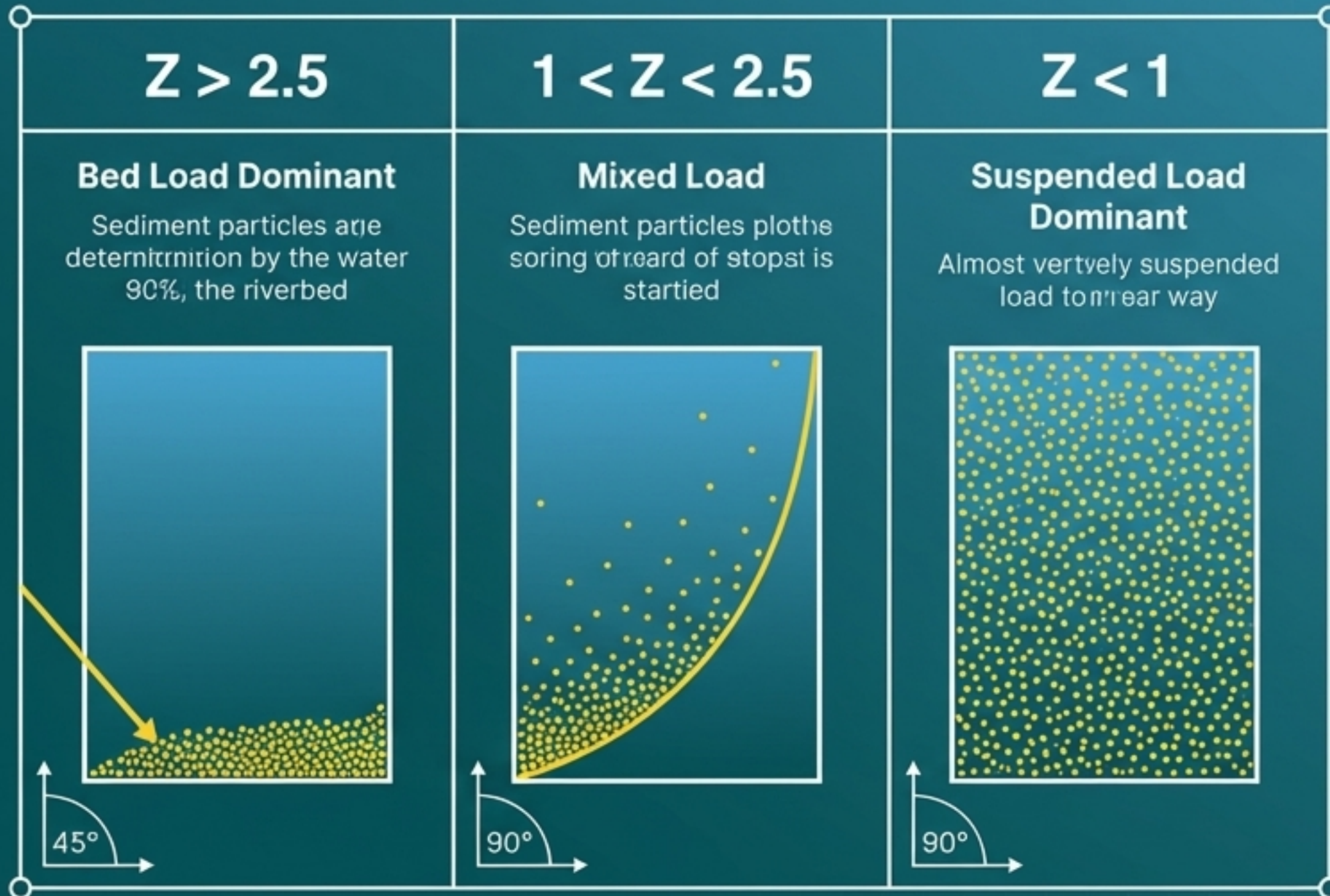
The Balance of Power

Helvetica Now Display
The Rouse Exponent. Dictates the shape of the vertical distribution.
Inter

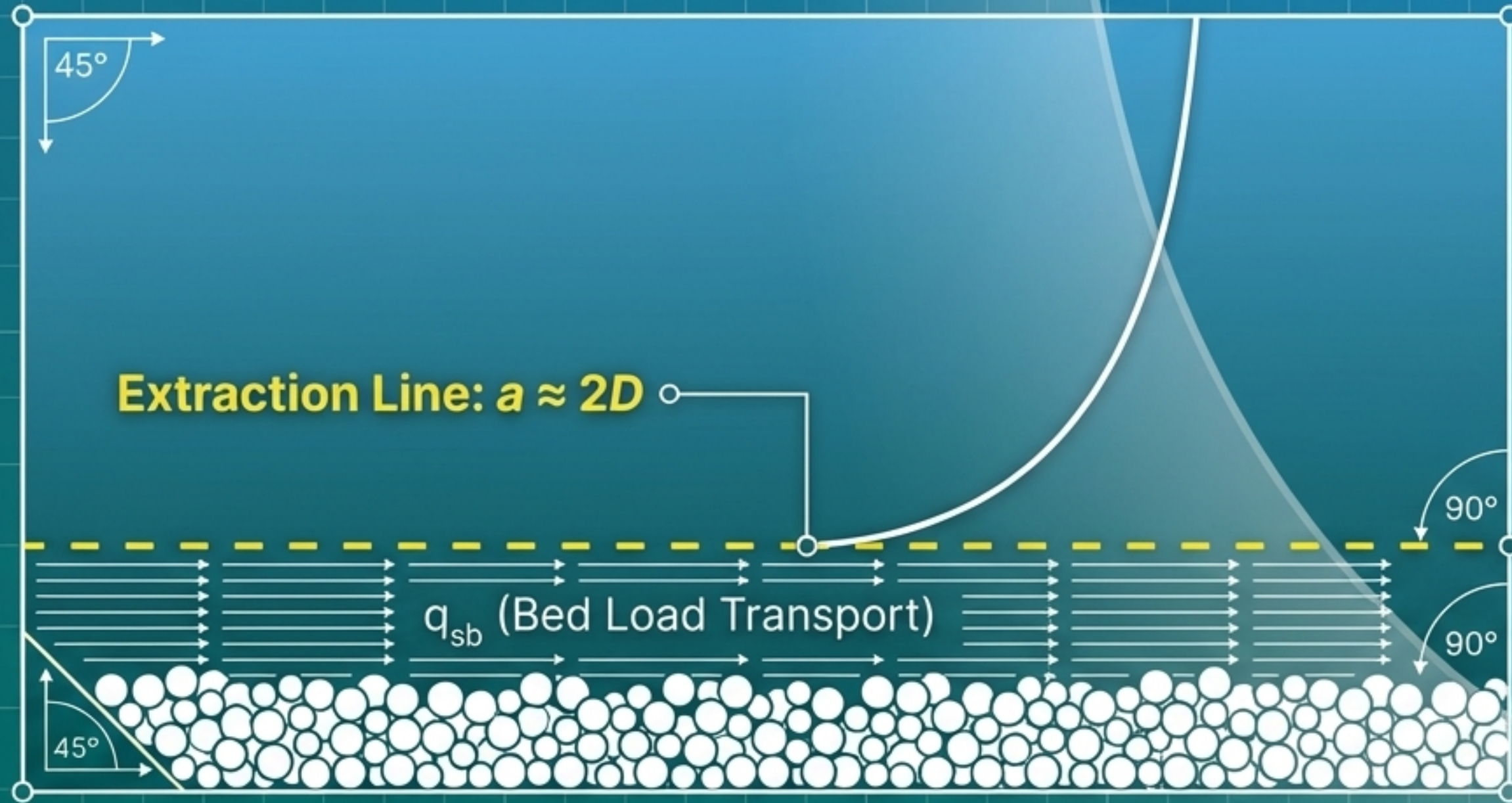
Decoding the Rouse Number (Z)

$$Z = \frac{V_s}{\kappa u_*}$$

Ratio of settling velocity to turbulent mixing intensity



Anchoring the Profile: The Boundary Condition (C_a)



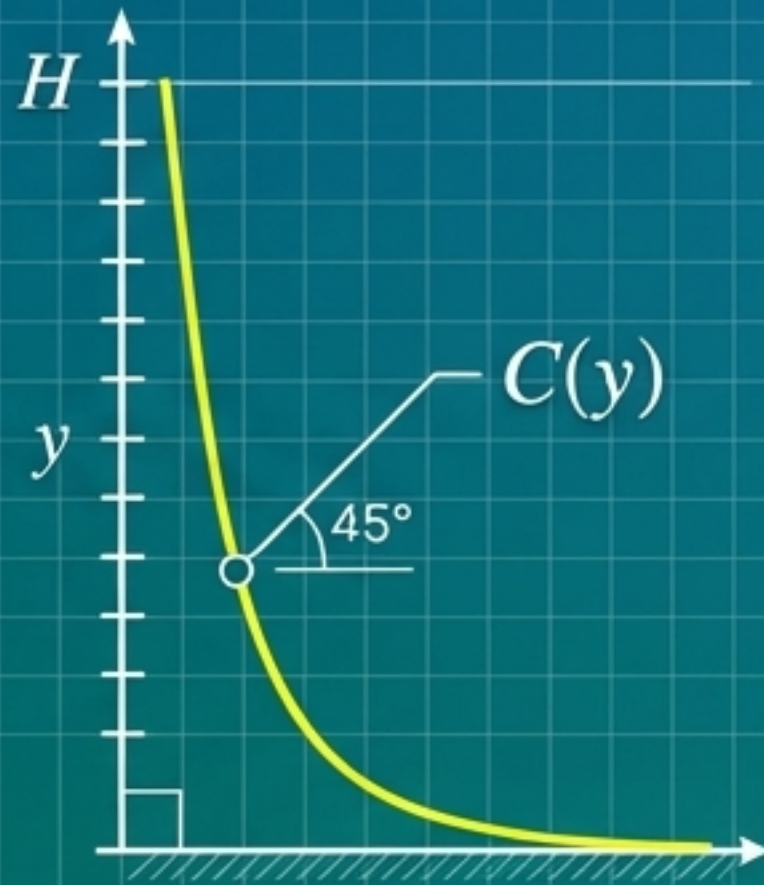
The Einstein Bridge

Einstein practically linked the unknown suspended anchor point to the known bed load transport rate below it.

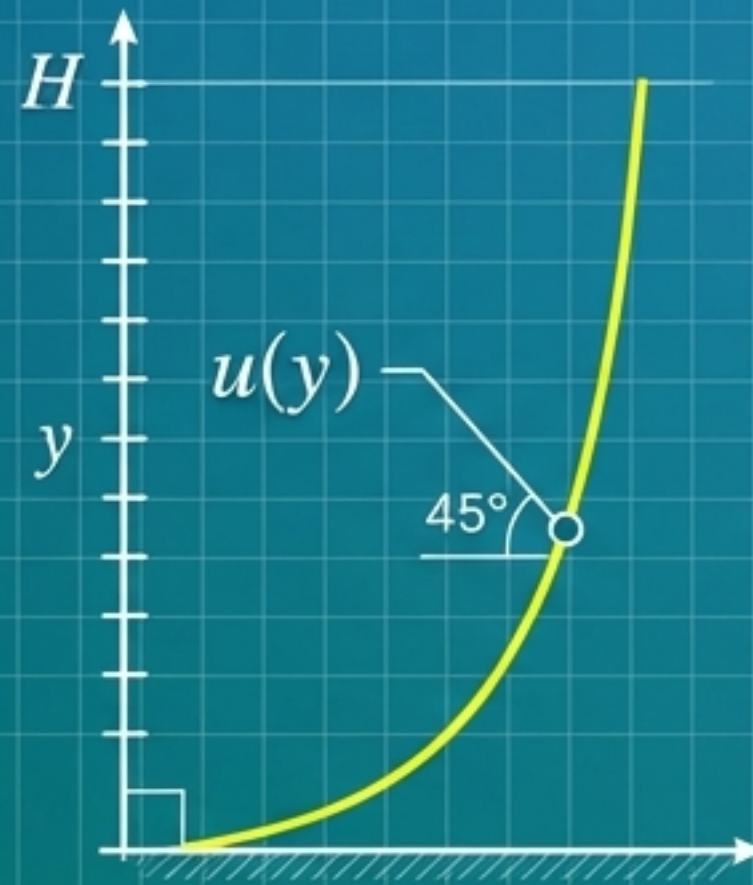
$$C_a = \frac{q_{sb}}{11.6 u'_* a}$$

The Master Synthesis: Transport Rate (q_{ss})

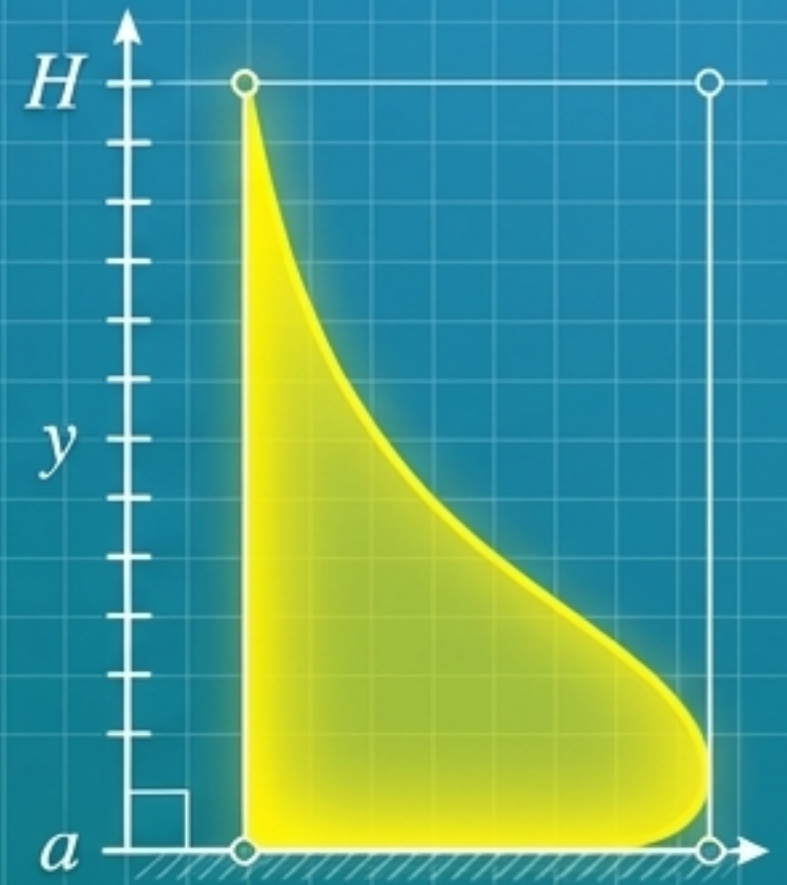
Concentration Profile



Log Velocity Profile

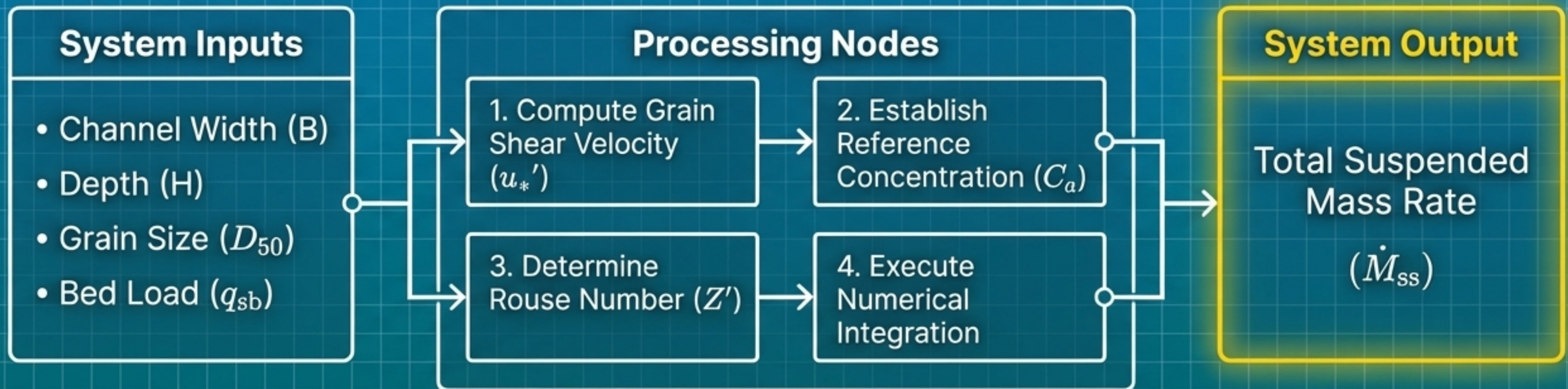


Transport Flux Integral



$$q_{ss} = \int_a^H C(y)u(y)dy$$

Engineering Application: The Calculation Pipeline



Physical laws dictate that these steps must be executed sequentially to solve for the transport capacity of a real-world channel.

Step 1 & 2: Shear & Reference Concentration

Parameters

Given Metrics:

$$H = 3.0 \text{ m}$$

$$D_{50} = 0.6 \text{ mm}$$

$$q_{sb} = 0.000095 \text{ m}^2/\text{s}$$

1. Grain Shear Velocity

$$u_*' = \sqrt{\tau'/\rho} = \sqrt{2.90/1000} = 0.0539 \text{ m/s}$$

$$u_*' = 0.0539 \text{ m/s}$$

Driving force isolated to the grain level.

2. Reference Concentration C_a

Using Einstein's relation,

$$C_a = \frac{q_{sb}}{11.6 u_*' a}$$

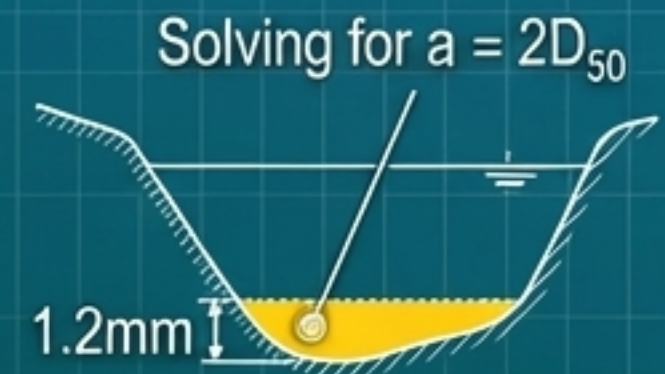
First compute the denominator:

$$11.6(0.0539)(0.0012) = 7.50 \times 10^{-4}$$

Then

$$C_a = \frac{9.5 \times 10^{-5}}{7.50 \times 10^{-4}} = 0.1267$$

$$C_a = 0.127$$



Step 3 & 4: Rouse Exponent & Integral Setup

3. The Rouse Exponent

$$Z' = \frac{V_s}{\kappa u_*'} = \frac{0.085}{0.40 \times 0.0539} = 3.95$$

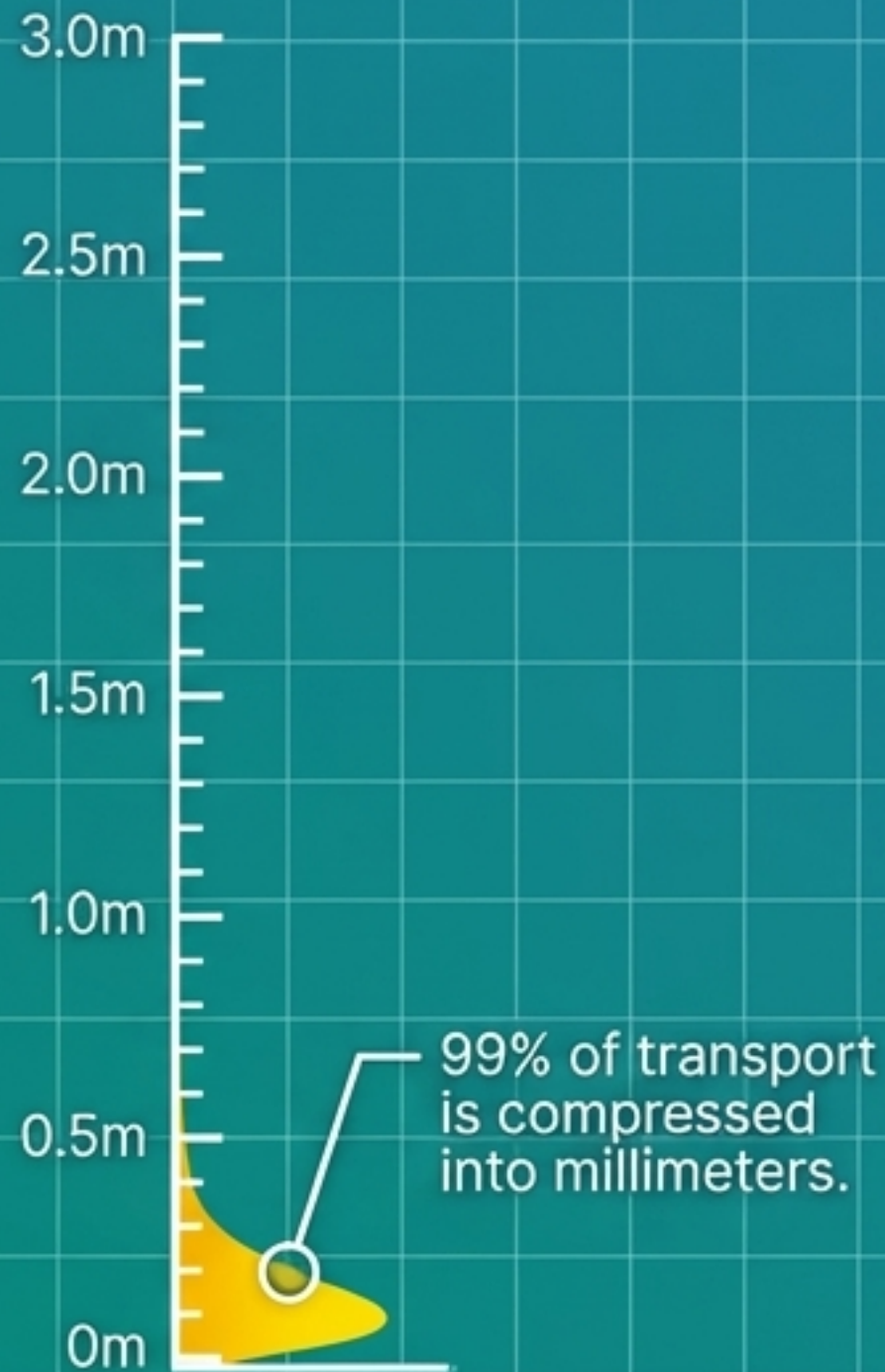
Insight: $Z' = 3.95$ indicates a highly bed-load dominant regime ($Z > 2.5$).

4. Populating the Master Integral

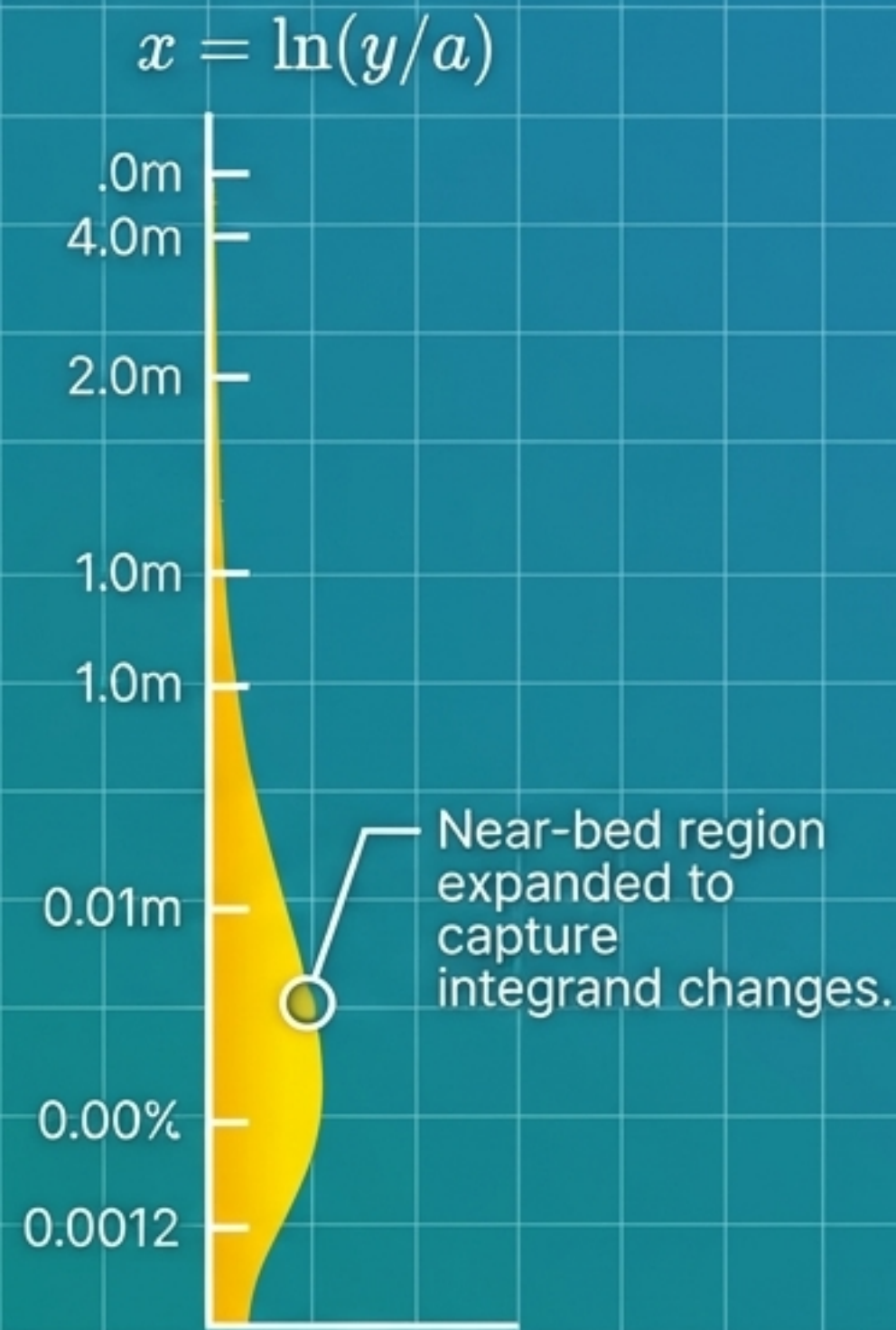
$$\frac{q_{ss}}{C_a} = \int_{0.0012}^{3.0} \left[\left(\frac{3-y}{y} \right) \times \frac{0.0012}{3-0.0012} \right]^{3.946} \times \frac{0.05385}{0.40} \ln \left(\frac{30y}{0.0015} \right) dy$$

The Integration Strategy: Why Logarithmic?

Linear Axis Distortion



Logarithmic Axis Expansion



Data Proof

y (m)	y/H	Integrand F(y)
0.0012	0.00040	0.4280
0.0020	0.00067	0.06279
0.0120	0.00400	2.82×10^{-5}
0.0300	0.01000	5.08×10^{-7}

The integrand becomes practically zero just 3cm above the bed.

Final Output: Total Suspended Load

Total Suspended Mass Rate

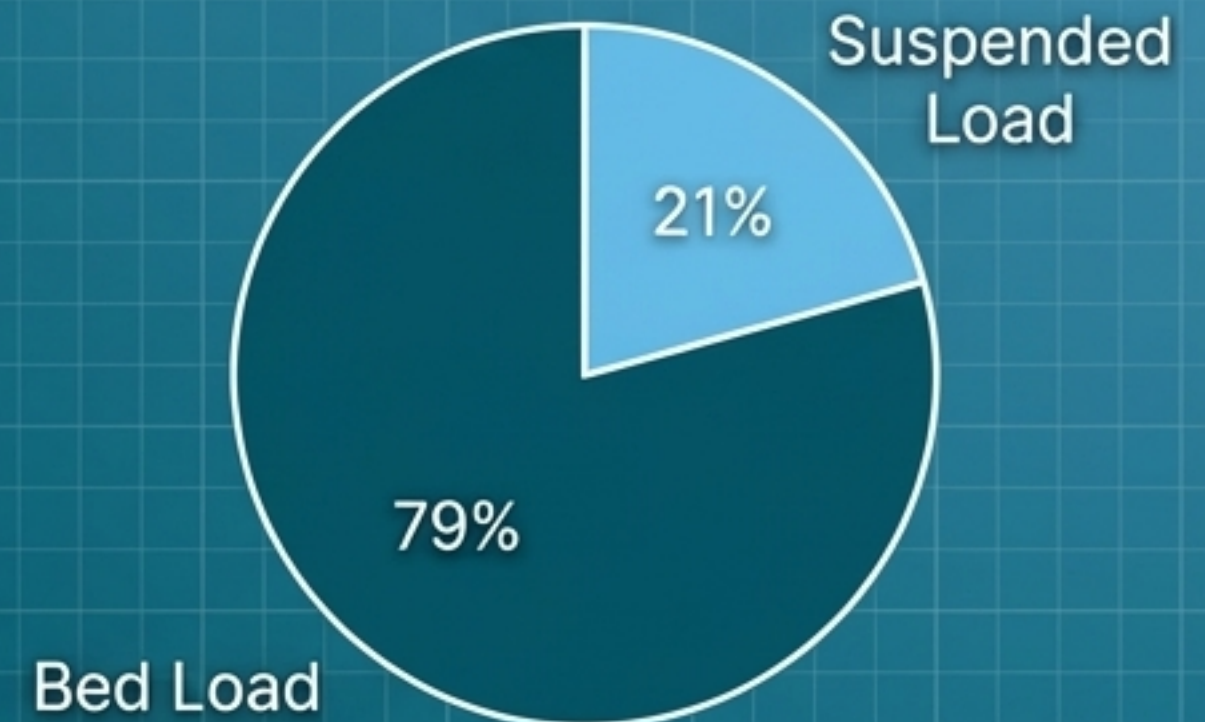
$$\dot{M}_{ss} = 3.24 \text{ kg/s}$$

Intermediate volumetric Step:

$$Q_{ss, \text{vol}} = 50 \times (2.44 \times 10^{-5}) = 1.22 \times 10^{-3} \text{ m}^3/\text{s}$$

Contextual Comparison

$$\frac{q_{ss}}{q_{sb}} = \frac{2.44 \times 10^{-5}}{9.5 \times 10^{-5}} \approx 0.26$$



Insight Validation: Suspended load is roughly 26% of bed load, perfectly validating our earlier Z' calculation of 3.95 which predicted a bed-load dominated river system.

Beyond Classical Theory: Reality & Limitations

Alternative Profiles Matrix

Rouse (Standard)	Lane-Kalinske (Simplified)	van Rijn (Advanced)
Most widely used. Based on log velocity and parabolic diffusivity.	Exponential decay . Simpler mathematically but less accurate near the bed.	Two-layer structure . Highly accurate boundary tracking for near-bed regions.

Real-World Limitations Checklist

- ! Nonequilibrium Transport**
Sediment supply and flow conditions vary continuously in nature.
- ! Sediment-Modified Turbulence**
High concentrations physically dampen turbulent mixing eddies.
- ! Variable Settling**
Particle interactions and clustering dynamically change settling velocities.

Classical equations provide the foundational blueprint, but modern hydrodynamic modeling requires adapting to the continuous, dynamic realities of the river.