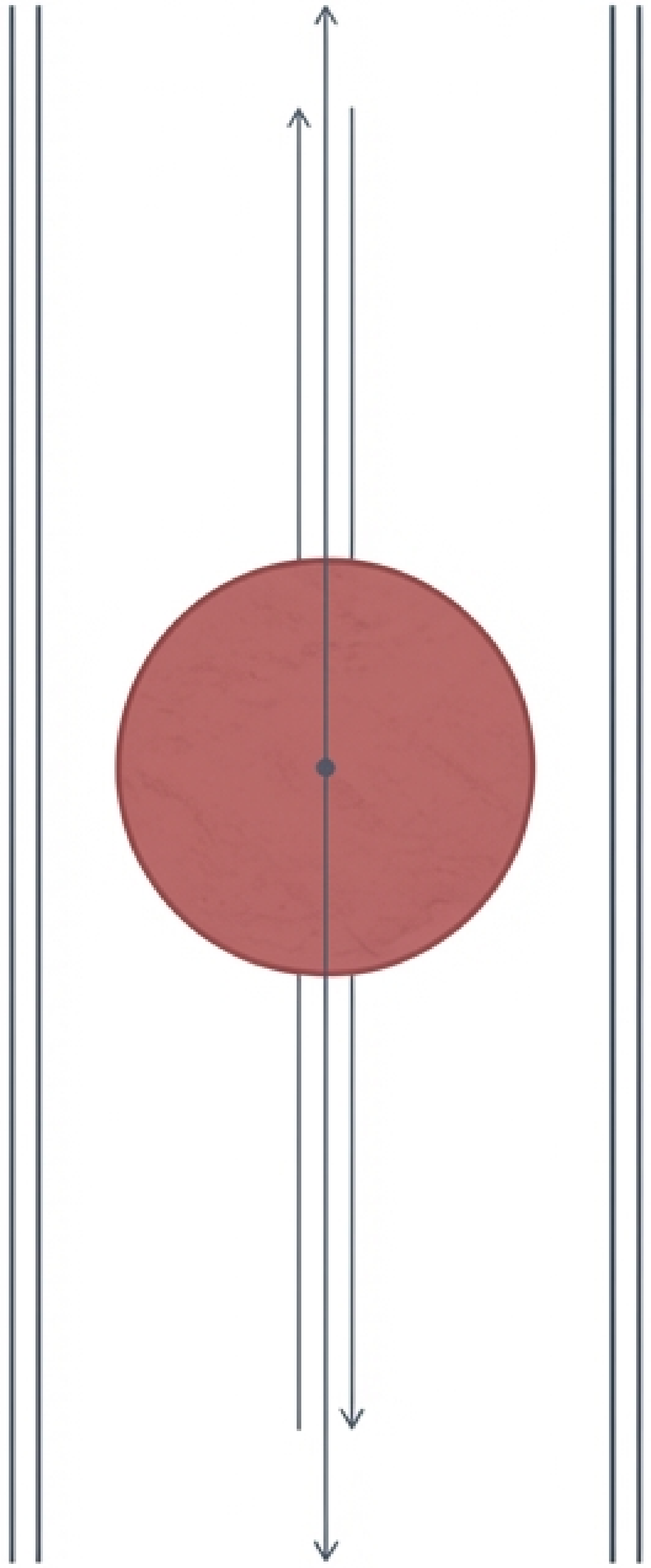
Particle Settling Velocity

The Physics of Sediment Transport, Force Balance, and Flow Regimes



Note: This deck covers the physical basis of individual particle settling (V_s) in clear, quiescent fluid.

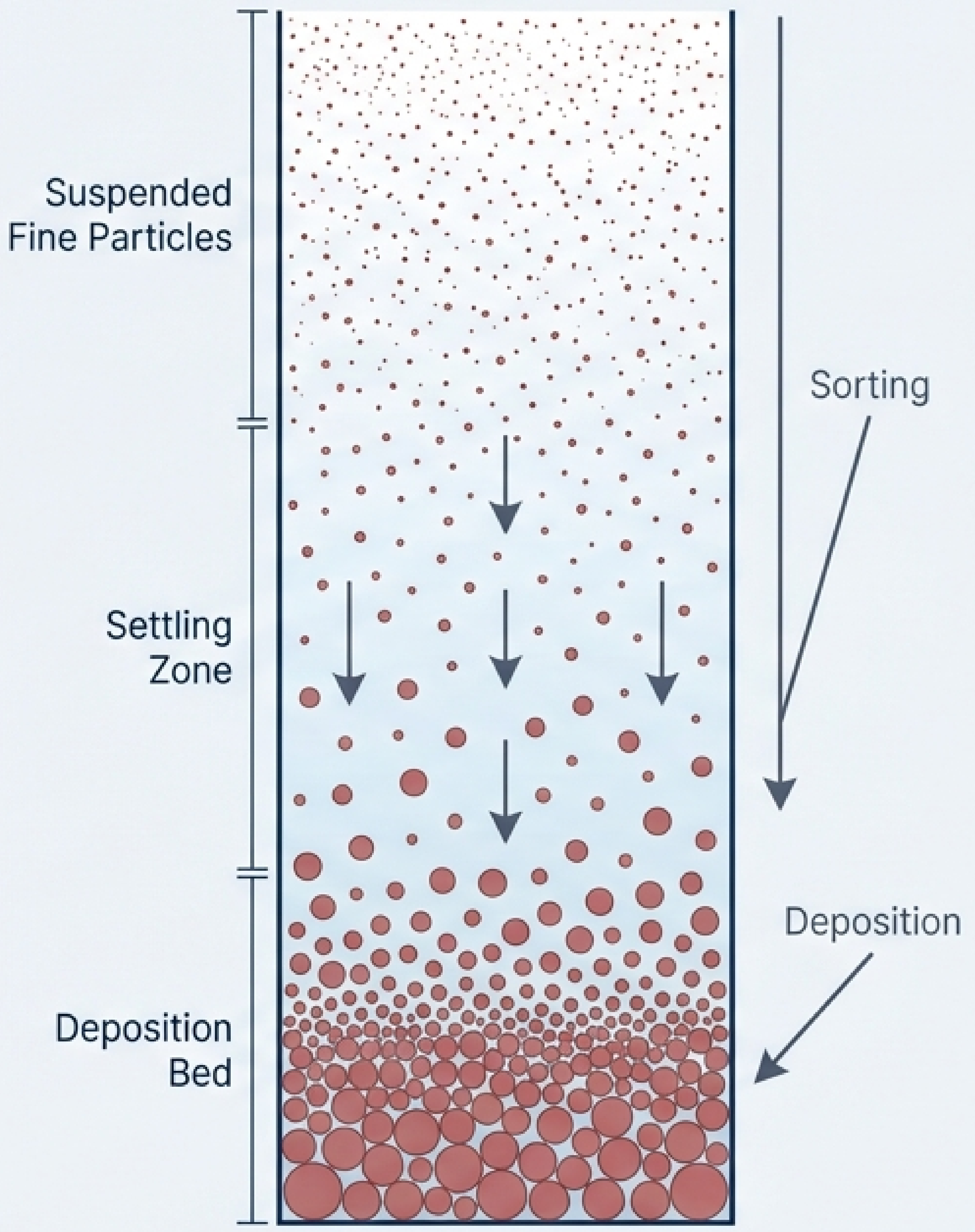
The DNA of Sediment Transport

Particle settling velocity (V_s) is the terminal vertical velocity attained by a sediment particle falling through a fluid under gravity. It is the governing property that determines:

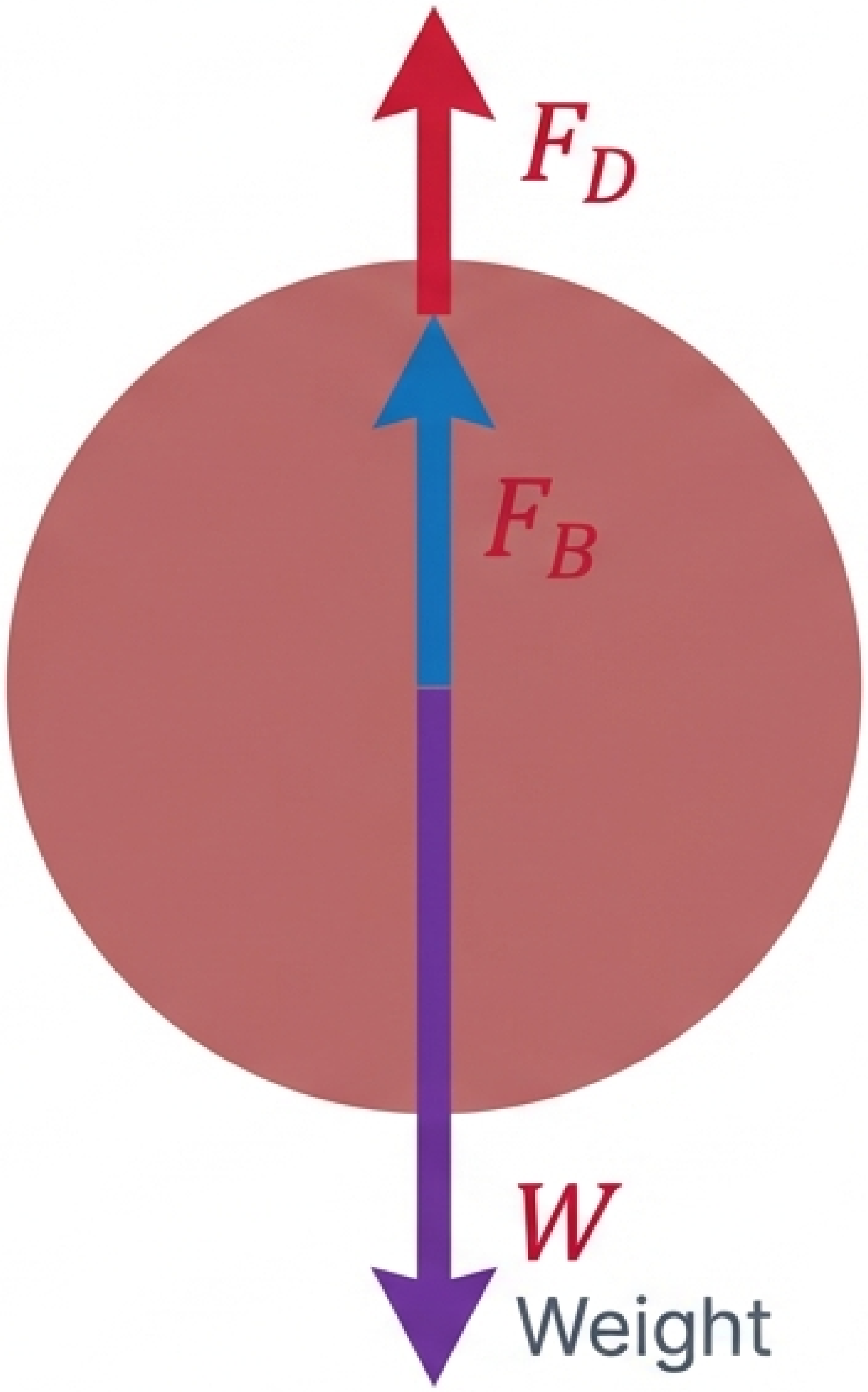
- Deposition versus suspension.
- Vertical sediment concentration profiles.
- Selective transport and grain-size sorting.
- Residence time of sediment in the water column.

Although settling appears to be a simple downward motion, it emerges from a delicate balance of competing forces whose importance shifts based on particle size and shape.

Vertical Sediment Settling and Sorting



The Force Balance

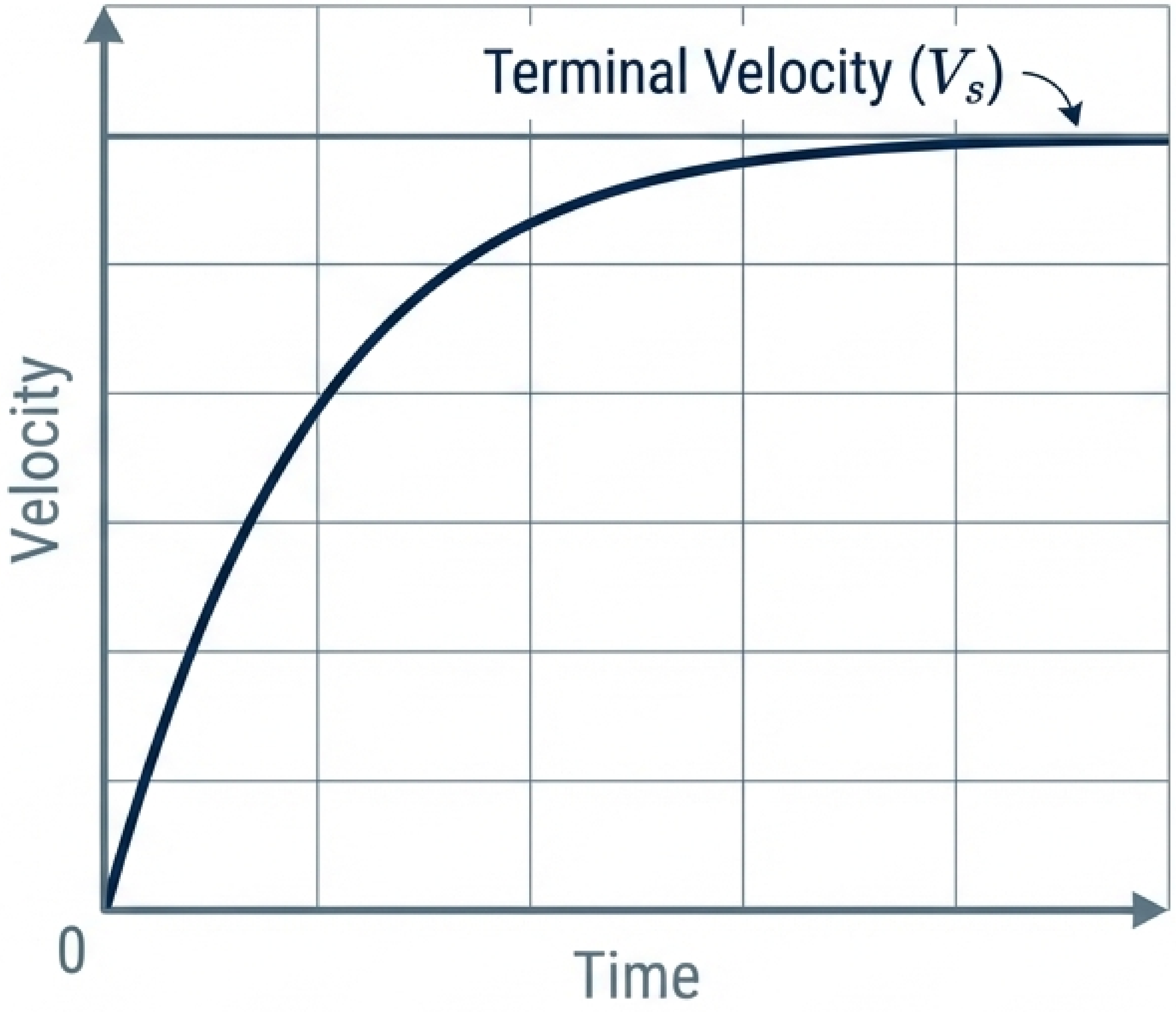


1. **Weight (W):** The gravitational pull on the particle mass ($W = \rho_s g V_p$).
2. **Buoyancy (F_B):** The upward force exerted by the displaced fluid ($F_B = \rho g V_p$).
3. **Drag (F_D):** The resistance of the fluid against motion ($F_D = 0.5 C_D \rho A V_s^2$).

Variables Key:

ρ_s (particle density), ρ (fluid density), V_p (particle volume), A (projected area), C_D (drag coefficient).

Reaching Equilibrium (Terminal Velocity)



Immediately after release, a particle accelerates because its submerged weight exceeds drag. As velocity increases, drag increases until it exactly balances the submerged weight.

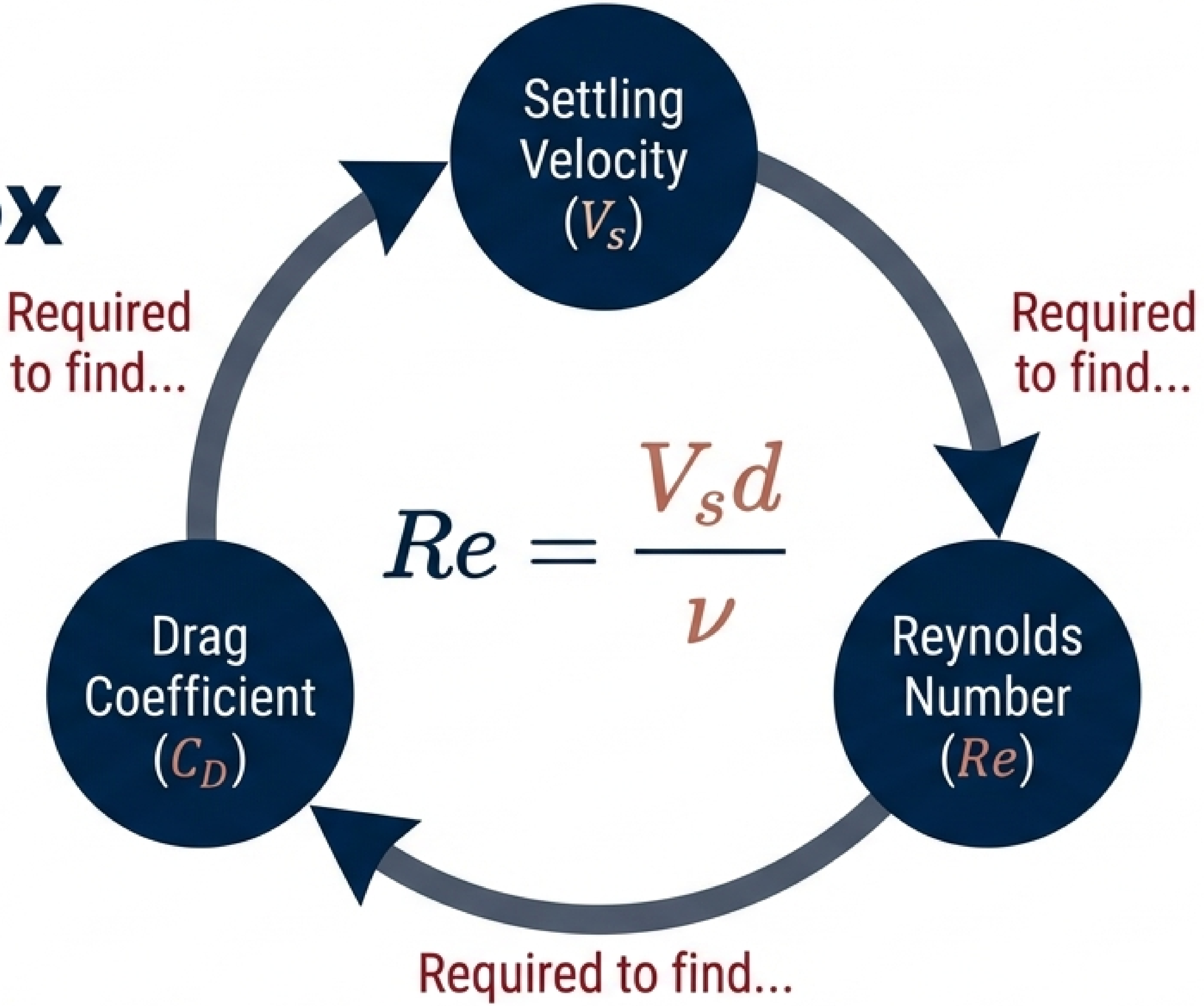
Force Balance Equation:

$$F_D = W - F_B$$

Expanded Form for a sphere:

$$0.5 C_D \rho \left(\frac{\pi d^2}{4}\right) V_s^2 = \left(\frac{\pi d^3}{6}\right)(\rho_s - \rho) g$$

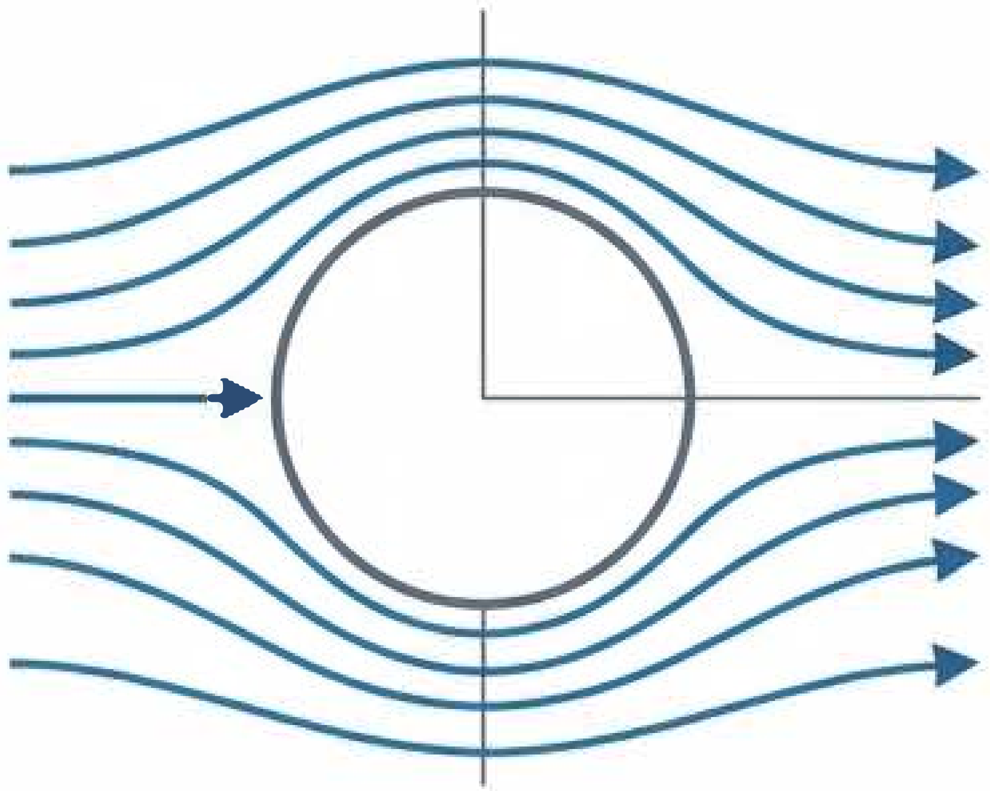
The Reynolds Number Paradox Paradox



Because Re depends on the unknown V_s, calculations are inherently regime-dependent and often require iteration.

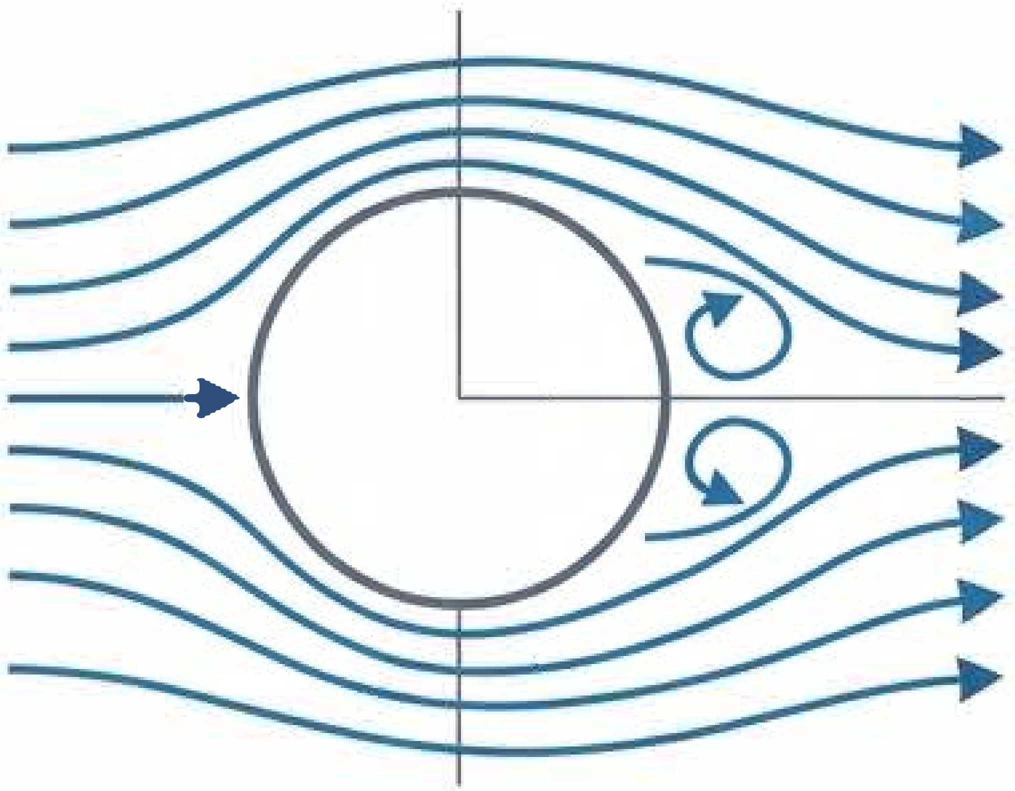
Flow Regimes define Drag Behavior

Low Re ($Re \lesssim 1$)



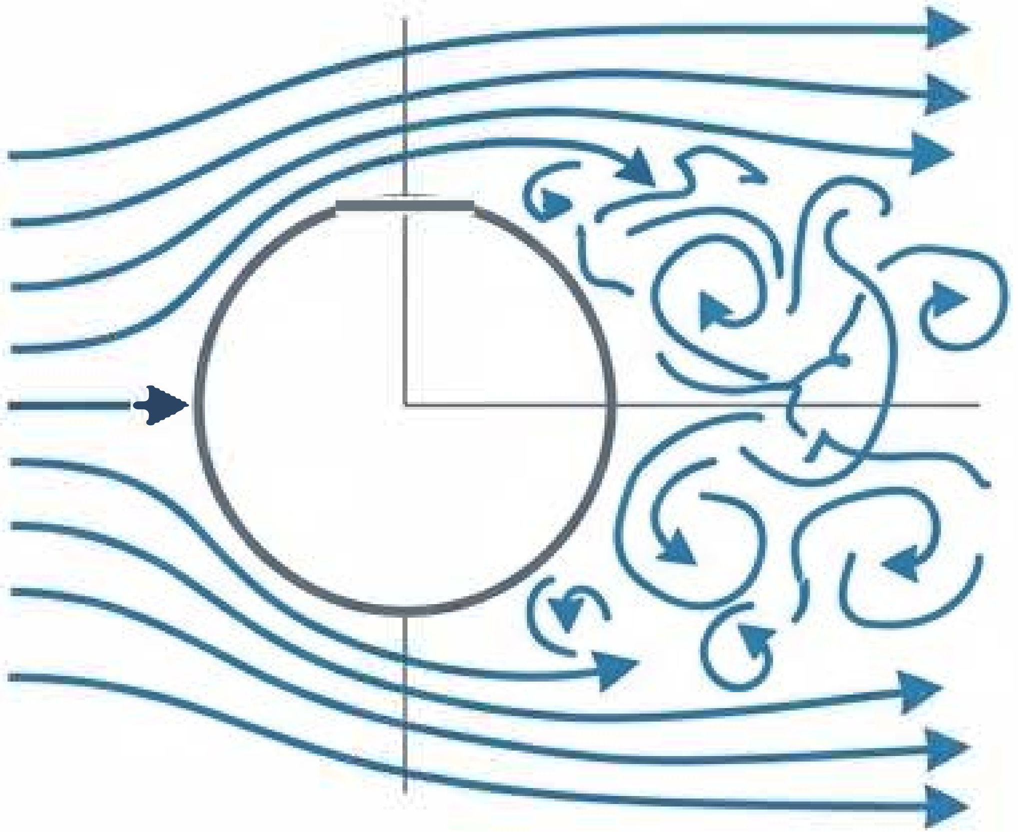
Stokes / Viscous Regime. Flow is fully laminar; boundary layers remain attached. Drag is dominated by viscous shear.

Transitional ($1 < Re < 1000$)



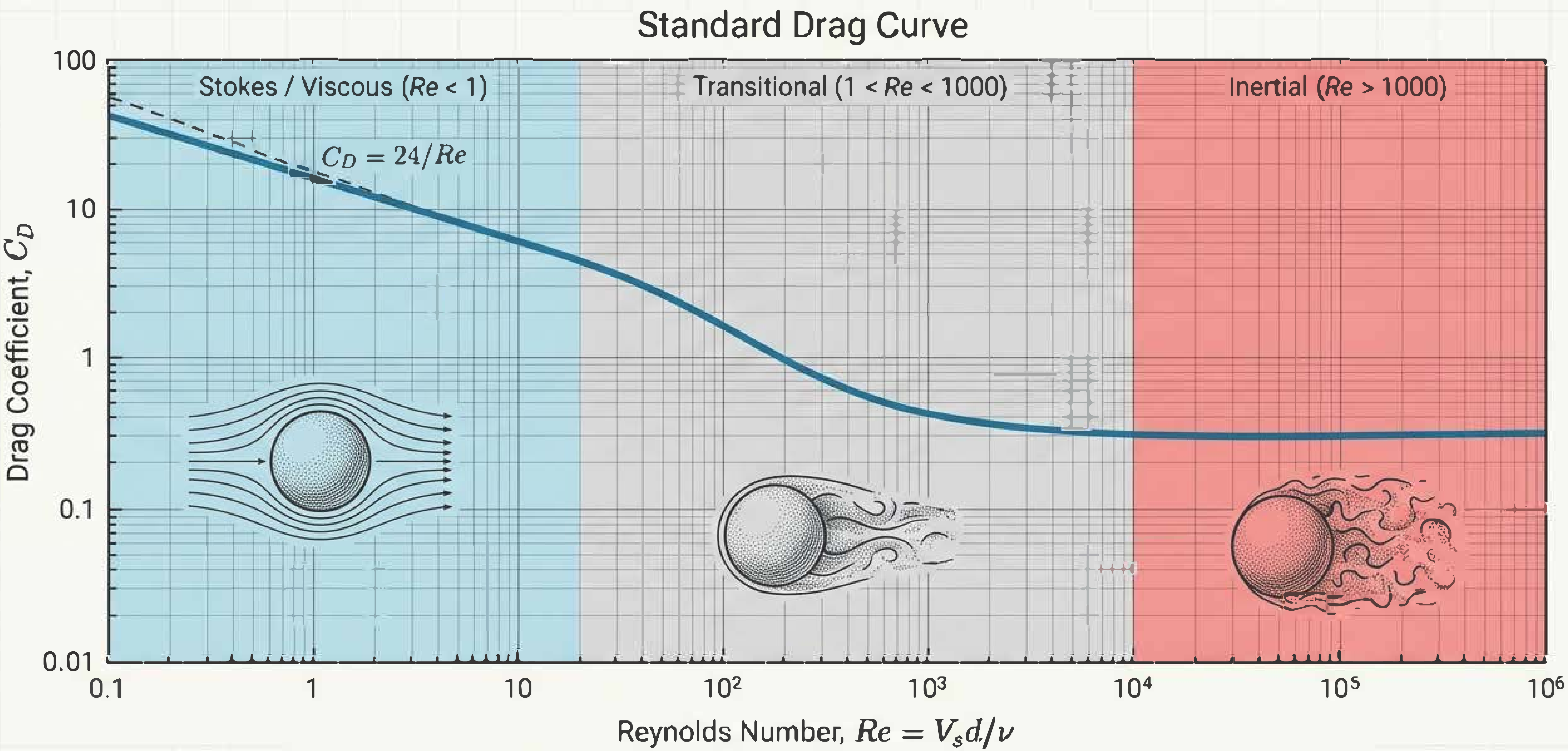
Flow separates from the surface; a steady wake forms. Drag transitions from viscous-dominated to pressure-dominated.

High Re ($Re \gtrsim 1000$)



Inertial Settling. Drag dominated by pressure (form) drag due to wake structure. C_D becomes approximately constant.

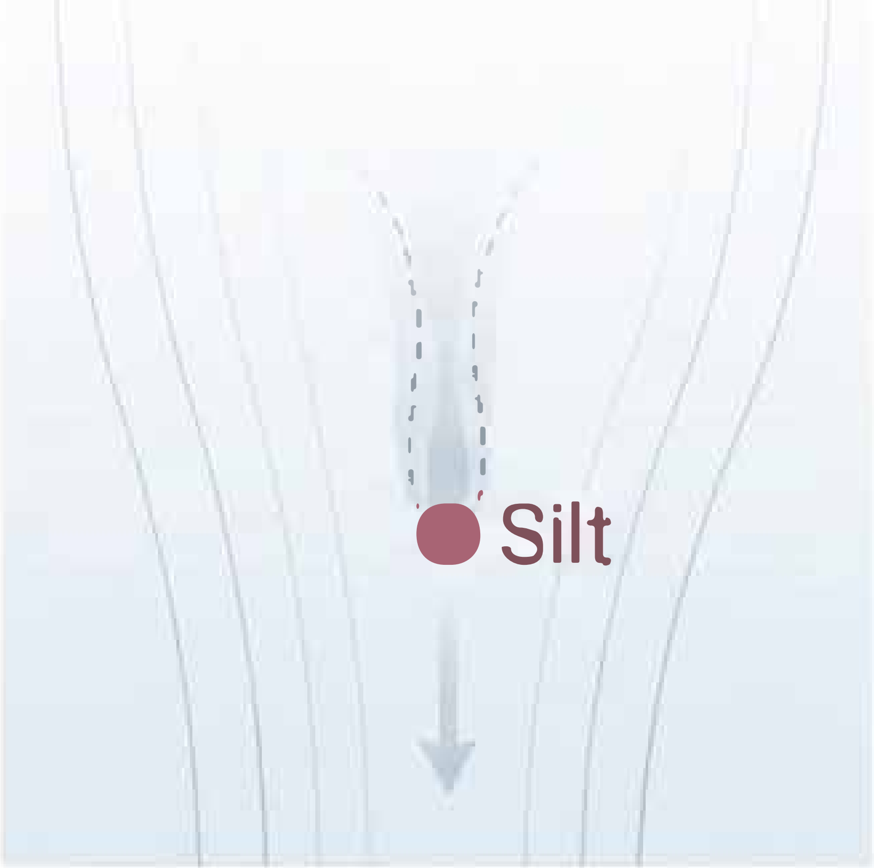
The Drag Coefficient Curve



The Analytical Solution: Stokes' Law

Valid only for $Re \lesssim 1$ (Silt and clay).

$$V_s = \frac{gd^2(\rho_s - \rho)}{18\mu}$$

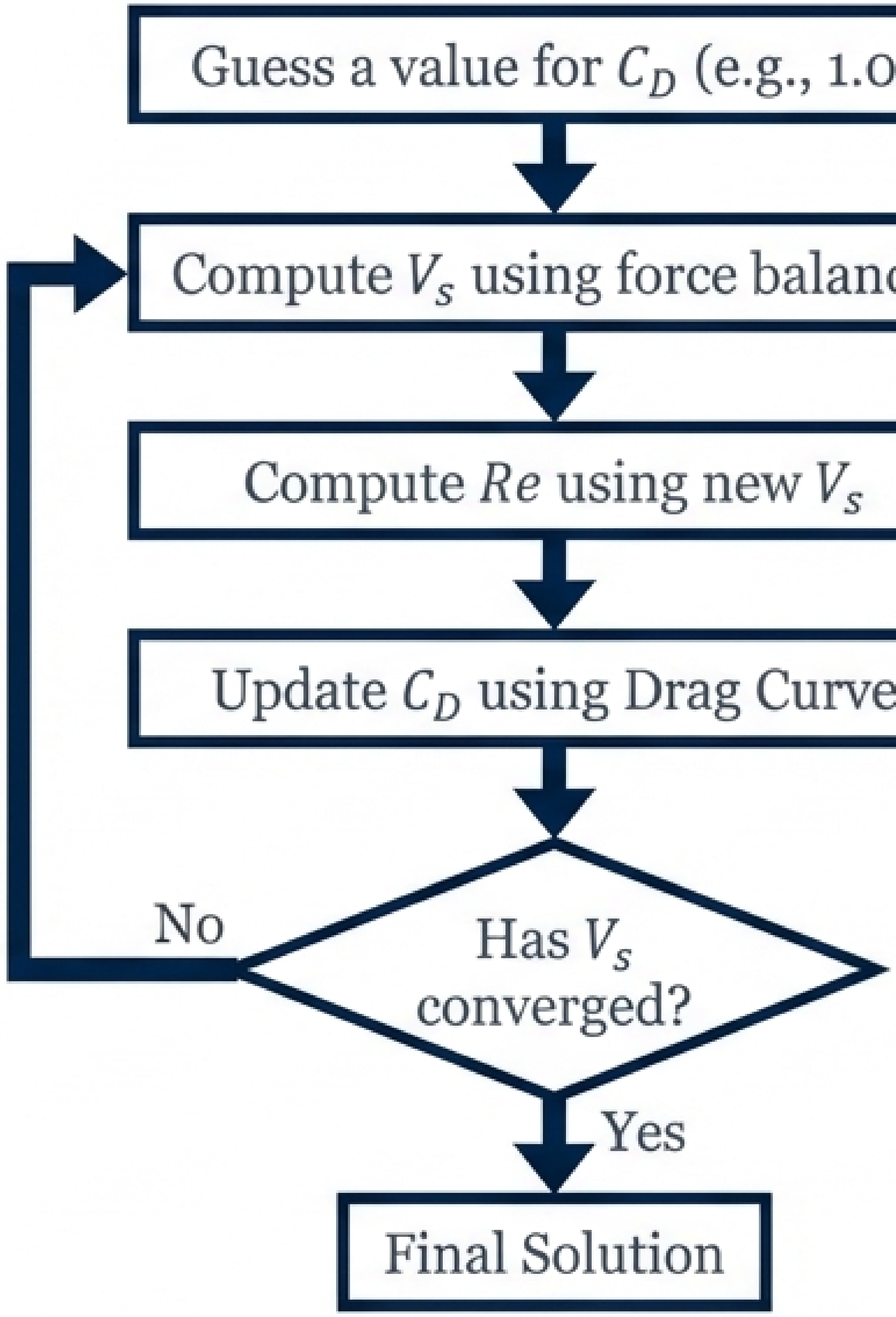


Key Characteristics:

- Velocity scales with the square of the diameter ($V_s \propto d^2$).
- Strong sensitivity to temperature (via viscosity μ).
- Provides a theoretical lower bound for settling velocity.

Beyond Stokes: The Iterative Method

For particles larger than silt ($Re > 1$), there is no simple formula because C_D changes.



Worked Example: Quartz Sphere in Water

Problem Statement: Compute terminal settling velocity for a spherical quartz particle in clear water at 20°C.

Given parameters:

- Particle Diameter (d): $1.0 \text{ mm} \ (1.0 \times 10^{-3} \text{ m})$
- Particle Density (ρ_s): 2650 kg/m^3
- Water Density (ρ): 1000 kg/m^3
- Kinematic Viscosity (ν): $1.0 \times 10^{-6} \text{ m}^2/\text{s}$
- Submerged Relative Density ($s-1$): 1.65

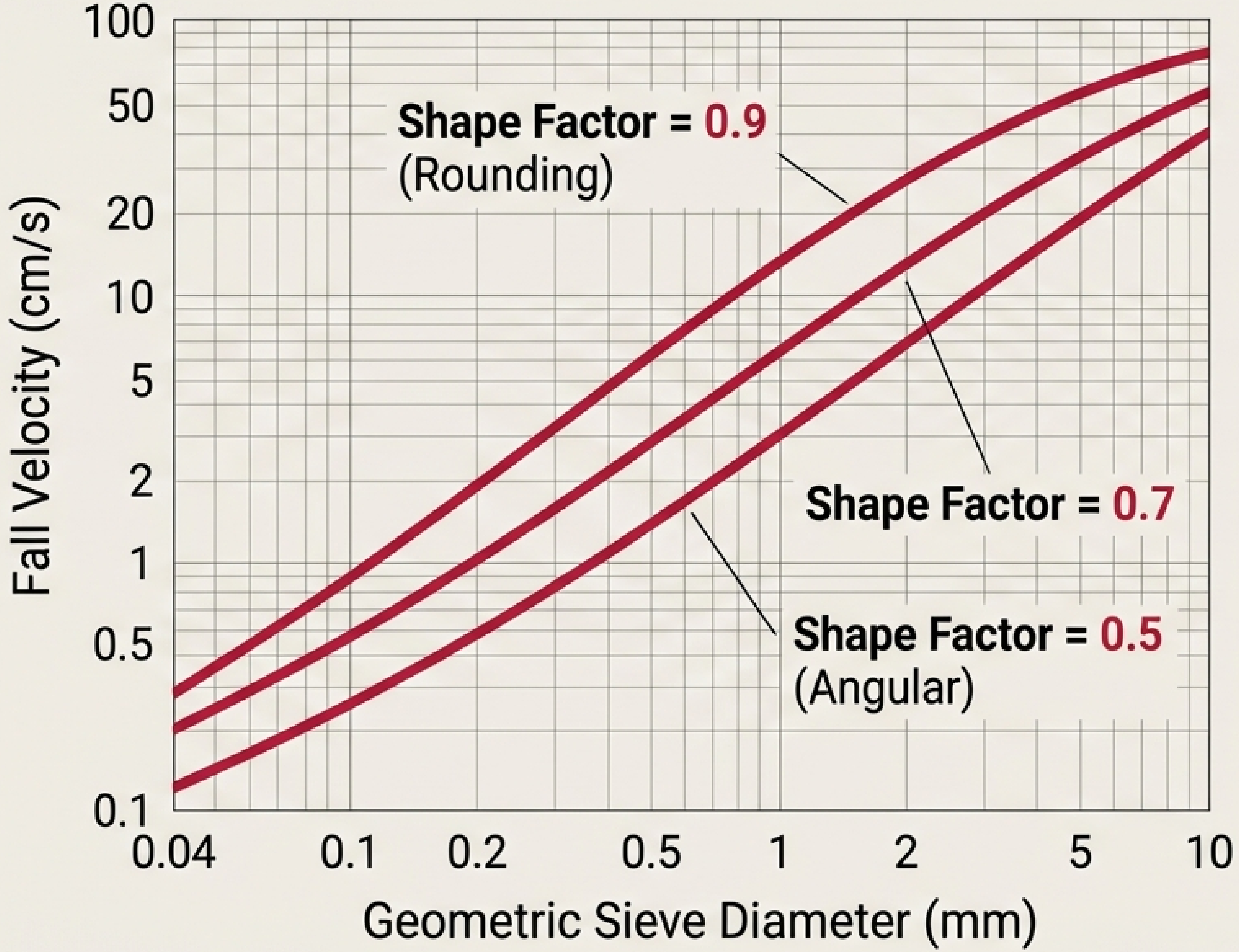
$$V_s = \sqrt{\frac{4gd(s-1)}{3C_D}}$$

Convergence in Three Steps

Iteration	Assumed C_D	Computed V_s (m/s)	Computed Re	Updated C_D
0 (Start)	1.00	0.1469	147	0.90
1	0.90	0.1548	155	0.88
2	0.88	0.1566	157	0.87
3 (Converged)	**0.87**	**0.158**	**158**	**0.87**

Final $V_s = 15.8\ \mathrm{cm/s}$. The final Reynolds number ($Re \approx 160$) confirms we are well outside the Stokes regime.

The Effect of Particle Shape



Reality Check: Natural sediment is rarely spherical.

- **Irregularity Increases Drag:** Angular or platy particles settle more slowly than spheres of the same mass.
- **Shape Factor (S.F.):** Engineers use S.F. to adjust the standard curves. **Lower S.F. = Lower Velocity.**

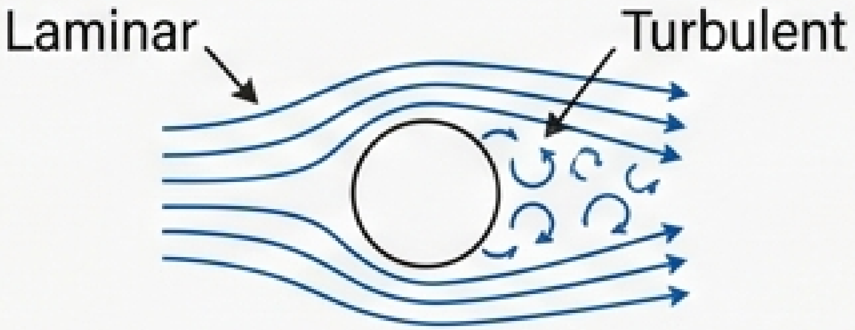
Governing Nondimensional Parameters

The Universal Framework

Reynolds Number (Re)

$$Re = \frac{V_s d}{\nu}.$$

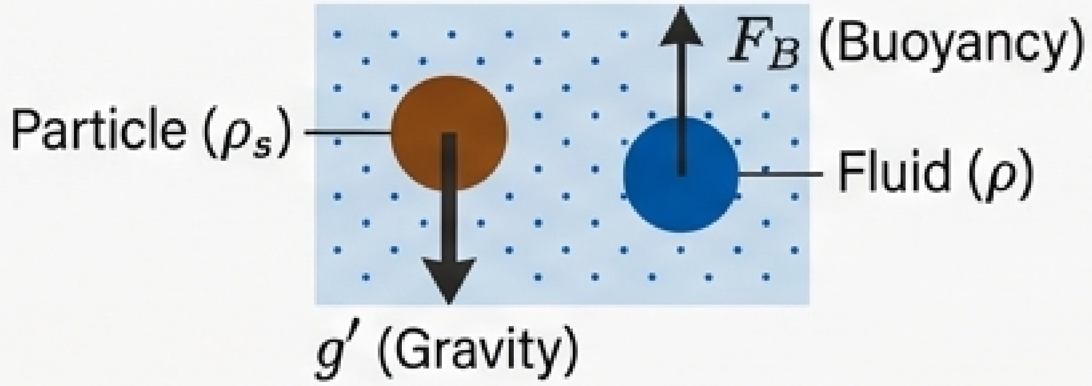
Defines the flow regime (Laminar vs. Turbulent wake).



Density Contrast ($\mathcal{R}$)

$$\mathcal{R} = \frac{\rho_s}{\rho} - 1$$

Drives the gravitational acceleration (g').



These two parameters, modified by Shape Factor, allow us to scale settling behavior across any fluid or particle type.

Scope and Practical Implications

Scope Assumptions

This lecture treated a single particle in clear, quiescent fluid.

Why It Matters

Errors in calculating V_s propagate directly into suspended load predictions, deposition rates, and morphodynamic modeling.

Real-World Factors Not Considered

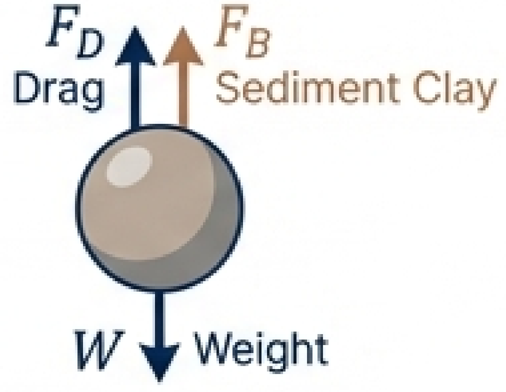
Hindered Settling: Sediment concentration effects (particles interfering with each other).

Turbulence: Interaction with ambient fluid turbulence.

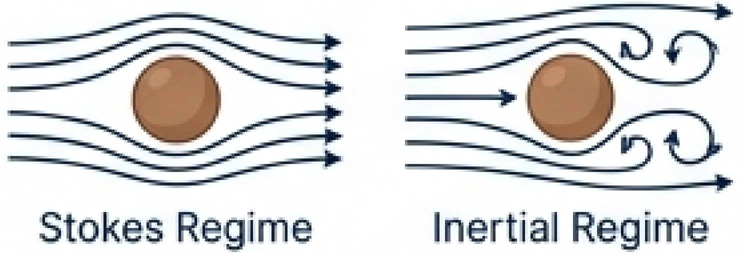
Flocculation: Cohesive sediments clumping together.

Key Takeaways

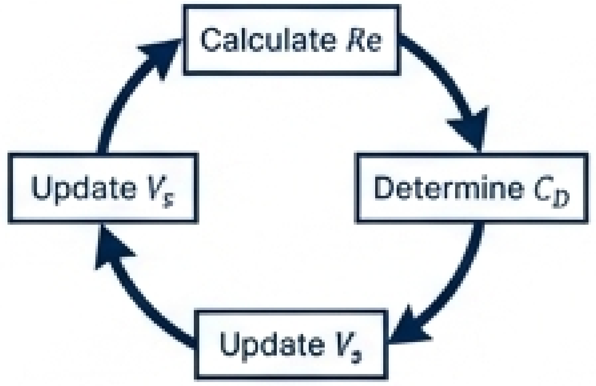
1. **Physics-Based:** Settling velocity arises from a precise force balance ($F_D = W - F_B$), not just empirical fitting.



2. **Regime-Dependent:** The flow regime (Stokes vs. Inertial) controls the physics; always identify which regime applies.



3. **Iteration is Key:** Outside of Stokes' Law ($Re > 1$), iterative calculation is required because Drag depends on Velocity.



4. **Shape Matters:** For natural sediment, particle shape is as critical as size—irregularity increases drag and slows deposition.