Particle Settling Velocity in Sediment Transport

Executive Summary

Particle settling velocity V_s is the terminal vertical velocity reached by a sediment particle falling through a fluid under gravity. It is a fundamental parameter in sediment transport because it determines:

- Whether particles remain in suspension or deposit
- Vertical concentration profiles
- Grain-size sorting
- Residence time in the water column

Settling is governed by a dynamic force balance between particle weight, buoyancy, and fluid drag. Because the drag coefficient C_D depends on the Reynolds number Re, and Re depends on V_s, settling velocity calculations are regime-dependent and typically require iteration.

- **Stokes' Law** provides an analytical solution for very small particles in laminar flow.
- Larger particles in transitional or inertia-dominated regimes require empirical drag relationships and account for **form drag** caused by flow separation.

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1. Physical Significance of Settling Velocity

Settling velocity is **not a material constant**. It emerges from the interaction between:

- **Particle properties:** size, shape, density
- **Fluid properties:** density, viscosity

In sediment transport, V_s governs:

- **Deposition vs. Suspension** – Whether particles settle out of the flow
- **Vertical Concentration Profiles** – Distribution through the water column
- **Selective Transport** – Grain-size sorting mechanisms
- **Residence Time** – Time before reaching the bed

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2. Fundamental Mechanics: Force Balance

For a single particle settling in still water, three vertical forces act:

2.1 Primary Forces

Force	Formula	Description
Weight W	$W = \rho_s g V_p$	Downward gravitational force
Buoyant Force F_B	$F_B = \rho g V_p$	Upward force from displaced fluid
Drag Force F_D	$F_D = \frac{1}{2} C_D \rho A V_s^2$	Resistance opposing motion

Where:

- ρ_s= particle density
- ρ= fluid density
- V_p= particle volume
- A= projected area

2.2 Terminal Velocity Concept

When released, the particle accelerates because:

$$W - F_B > F_D$$

As velocity increases, drag increases until equilibrium:

$$F_D = W - F_B$$

At this point, acceleration ≈ 0 and terminal velocity $V_s$is reached.

For a spherical particle of diameter d:

$$\frac{1}{2} C_D \rho \frac{\pi d^2}{4} V_s^2 = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 (\rho_s - \rho) g$$

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3. Hydrodynamic Regimes and Reynolds Number

$$Re = \frac{V_s d}{\nu}$$

Where:

- ν= kinematic viscosity

Because Redepends on unknown V_s, determining regime is essential.

3.1 Low Reynolds Number (Stokes Regime, $Re \lesssim 1$)

- Fully laminar flow
- Boundary layer attached
- No wake formation
- Drag dominated by viscous shear

Drag coefficient:

$$C_D = \frac{24}{Re}$$

Stokes' Law:

$$V_s = \frac{(\rho_s - \rho) g d^2}{18\mu}$$

Where μ= dynamic viscosity.

Key property:

$$V_s \propto d^2$$

Applicable to clay and fine silt.

3.2 Transitional Regime ($1 < Re < 10^3$)

- Flow separation begins
- Steady wake forms
- Drag transitions from viscous to pressure-dominated

C_D must be obtained from experimental curves.

3.3 High Reynolds Number ($Re \gtrsim 10^3$)

- Drag dominated by **form drag**
- Controlled by pressure differences
- $C_D \approx$ order 1
- Weak viscosity dependence

Drag crisis (smooth spheres only):

$$Re \approx 2 \times 10^5$$

Rare for natural sediment.

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4. Physical Origin of Form Drag

Form drag arises from:

1. Fluid acceleration around particle sides
2. Pressure reduction along sides
3. Flow separation
4. Low-pressure wake formation

Pressure difference between front and rear surfaces produces net resisting force.

This explains the rapid increase in drag after separation.

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5. Particle Shape and Natural Sediments

Natural sediment is rarely spherical.

Effects:

- Increased drag
- Slower settling
- Angular/platy particles (e.g., mica) settle more slowly

Shape corrections are introduced through:

- Modified C_D
- Shape factors (S.F.)

Common quartz sand S.F. values:

- 0.5
- 0.7
- 0.9

Temperature Sensitivity

Settling velocity depends on temperature through viscosity changes, especially in the Stokes regime.

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6. Iterative Calculation Procedure

Outside the Stokes regime, no closed-form solution exists.

Because:

$$V_s \leftrightarrow C_D \leftrightarrow Re$$

are interdependent.

6.1 Iteration Steps

1. Write force balance:

$$V_s = \sqrt{\frac{4gd(s-1)}{3C_D}}$$

where $s = \rho_s/\rho$

2. Assume initial C_D(e.g., 1.0)
3. Compute V_s
4. Compute Re
5. Update $C_D$from C_D–Re curve
6. Repeat until convergence (<1% change)

6.2 Worked Example: 1.0 mm Quartz Sphere

Given:

- $d = 1.0$ mm
- $\rho_s = 2650$ kg/m^3
- $\rho = 1000$ kg/m^3
- $\nu = 1.0 \times 10^{-6}$ m^2/s

Pre-computed:

$$V_s = \sqrt{\frac{0.021582}{C_D}}$$

Iteration:

- **Initial** $C_D = 1.0$
  - $V_s = 0.1469$m/s
  - $Re = 147$
- **Iteration 1:** $C_D \approx 0.90$
  - $V_s = 0.1548$m/s
  - $Re = 155$
- **Iteration 2:** $C_D \approx 0.88$
  - $V_s = 0.1566$m/s
  - $Re = 157$

Converged:

$V_s \approx 0.158$ m/s(15.8 cm/s)
$Re \approx 160$

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7. Framework and Scope

Settling behavior governed by:

- Reynolds number Re
- Density contrast

$$\mathcal{R} = \frac{\rho_s}{\rho} - 1$$

7.1 Assumptions

- Single particle
- Clear, still water
- Steady terminal conditions

7.2 Excluded Real-World Effects

- Hindered settling (high concentration)
- Turbulence–particle interaction
- Flocculation (cohesive sediment)
- Acceleration phase

Engineering Significance

Errors in settling velocity directly propagate into:

- Suspended load prediction
- Deposition rate estimation
- Morphodynamic modeling

Correct interpretation is essential for hydraulic and environmental engineering applications.