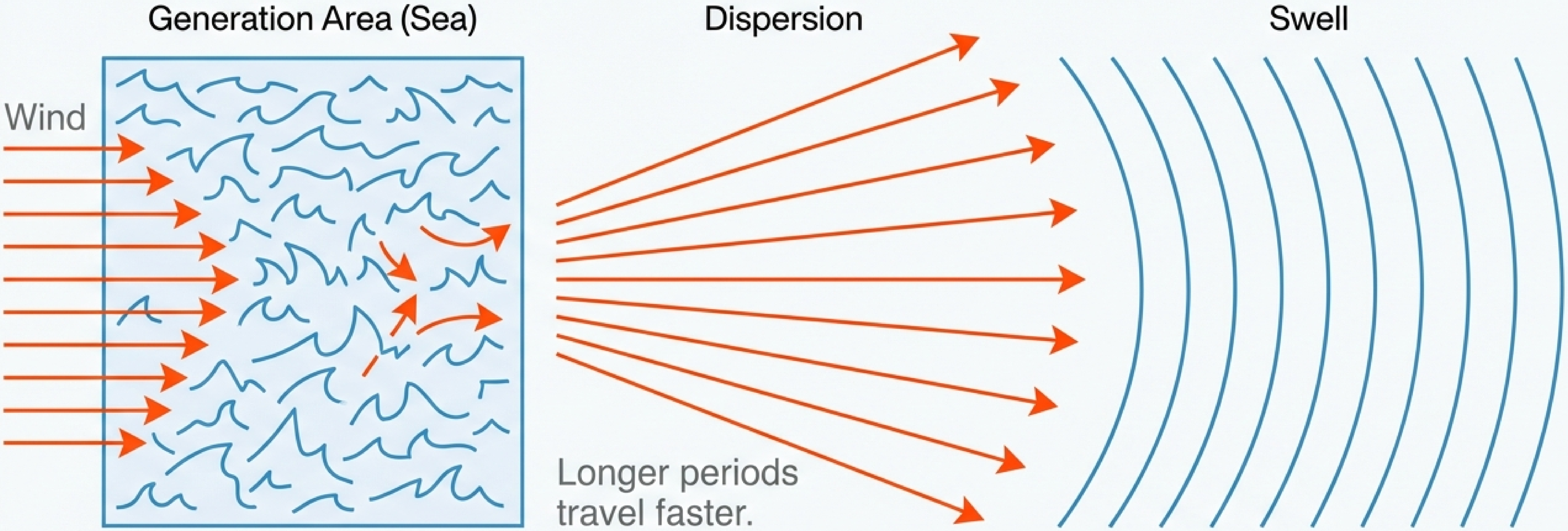
# The Physics of Coastal Waves

## From Deep Water Genesis to Nearshore Transformation

Context: The Journey of Energy. Following the lifecycle of a wave from wind-driven generation, through the governing laws of fluid dynamics, to the physical transformation upon encountering the coast.

# Genesis: Wave Generation and Dispersion



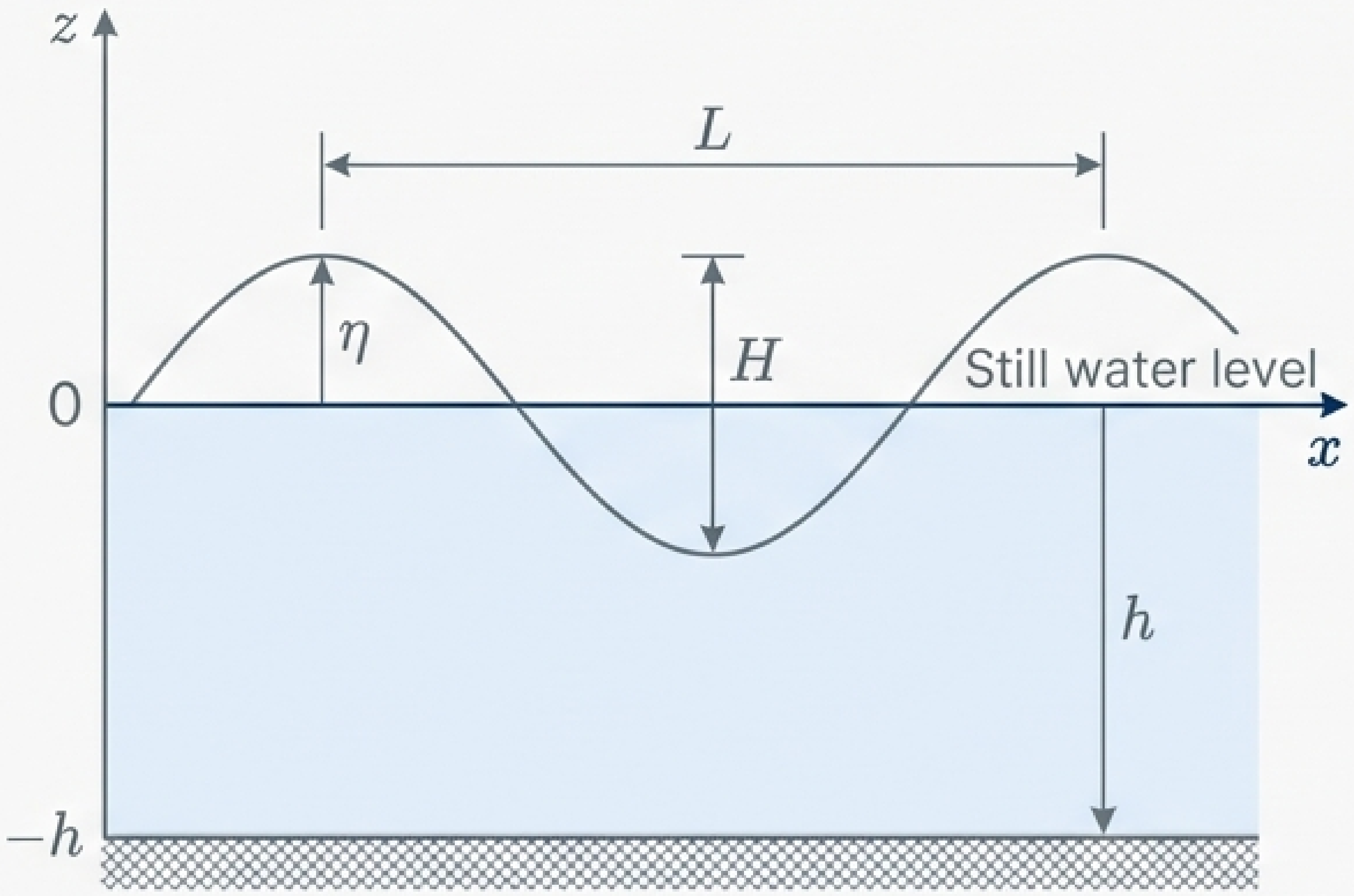

**Energy Transfer**: Wind stress on the surface creates chaotic, short-crested waves.

**Sorting**: Waves sort by period as they travel. Long waves outrun short ones.

**Propagation**: The result is organized "Swell" capable of crossing oceans.

# The Governing Laws: Small Amplitude Wave Theory

## Definition Sketch




## **Assumptions** required for Linear Theory

**Inviscid**
No internal friction/viscosity

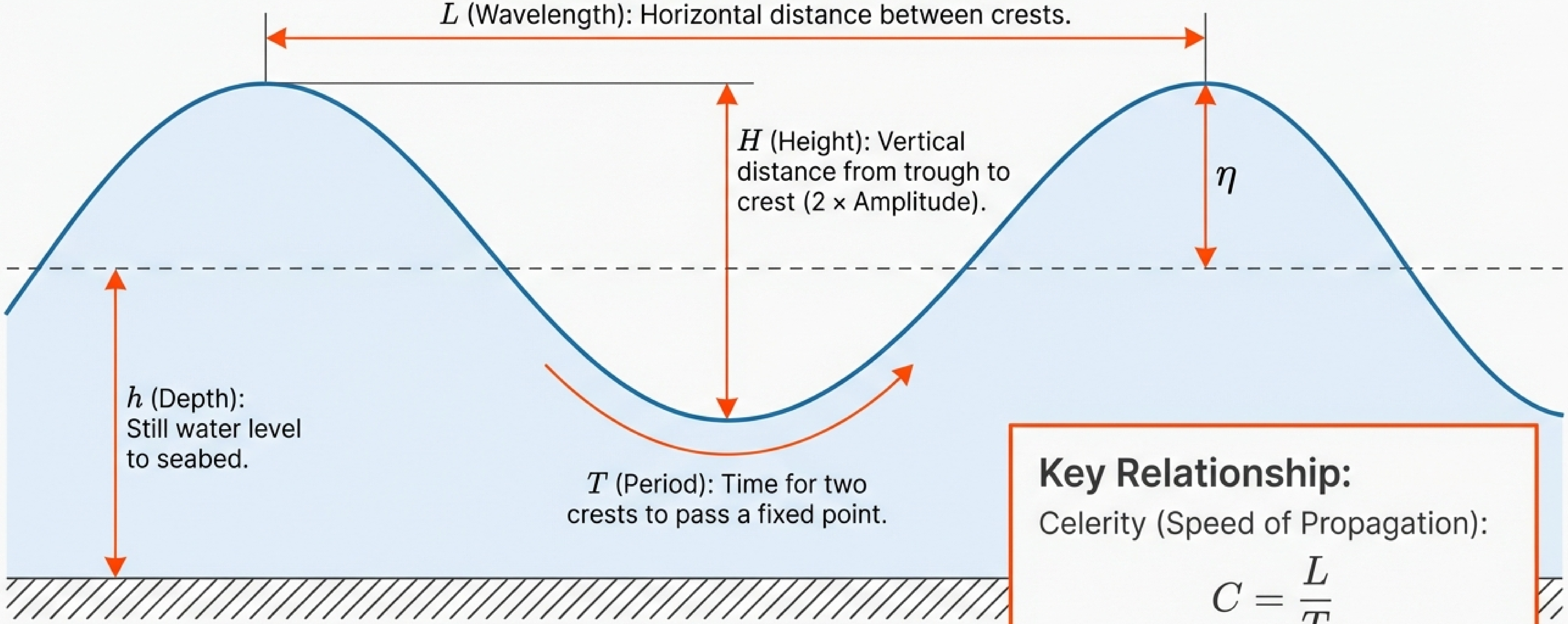
$\rho$=const
**Incompressible**
Constant density $\rho$

**Irrotational**
Particles do not spin, $\nabla \times V = 0$

**Small Amplitude**
$H \ll L$ and $H \ll h$

Laplace's Equation: $\nabla^2\phi = \dfrac{\partial^2\phi}{\partial x^2} + \dfrac{\partial^2\phi}{\partial z^2} = 0$

# The Anatomy of a Wave




**Key Relationship:**

Celerity (Speed of Propagation):

$$C = \frac{L}{T}$$

# The Dispersion Relation: How Depth Dictates Speed

$$\omega^2 = gk\tanh(kh)$$

## Deep Water ($h > L/2$)

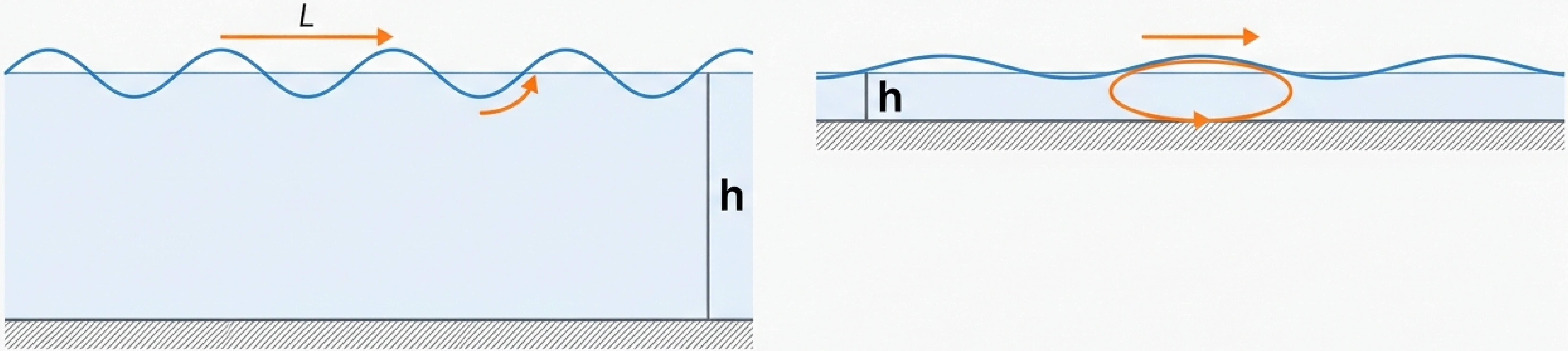
Speed depends on Period ($T$). Depth is irrelevant.

$C_0 \approx 1.56T$

## Shallow Water ($h < L/20$)

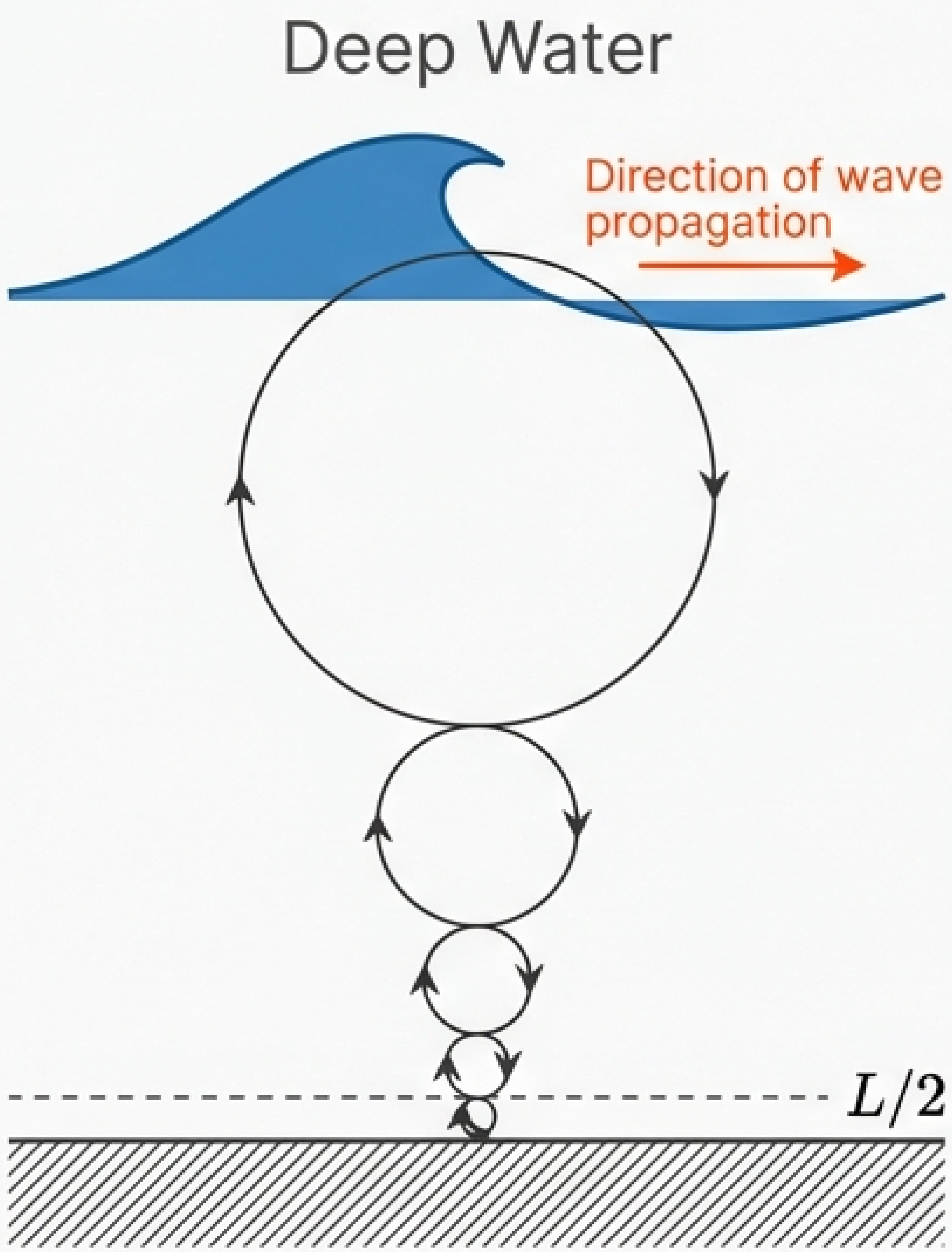
Speed depends on Depth ($h$). Wavelength is irrelevant.

$C = \sqrt{gh}$

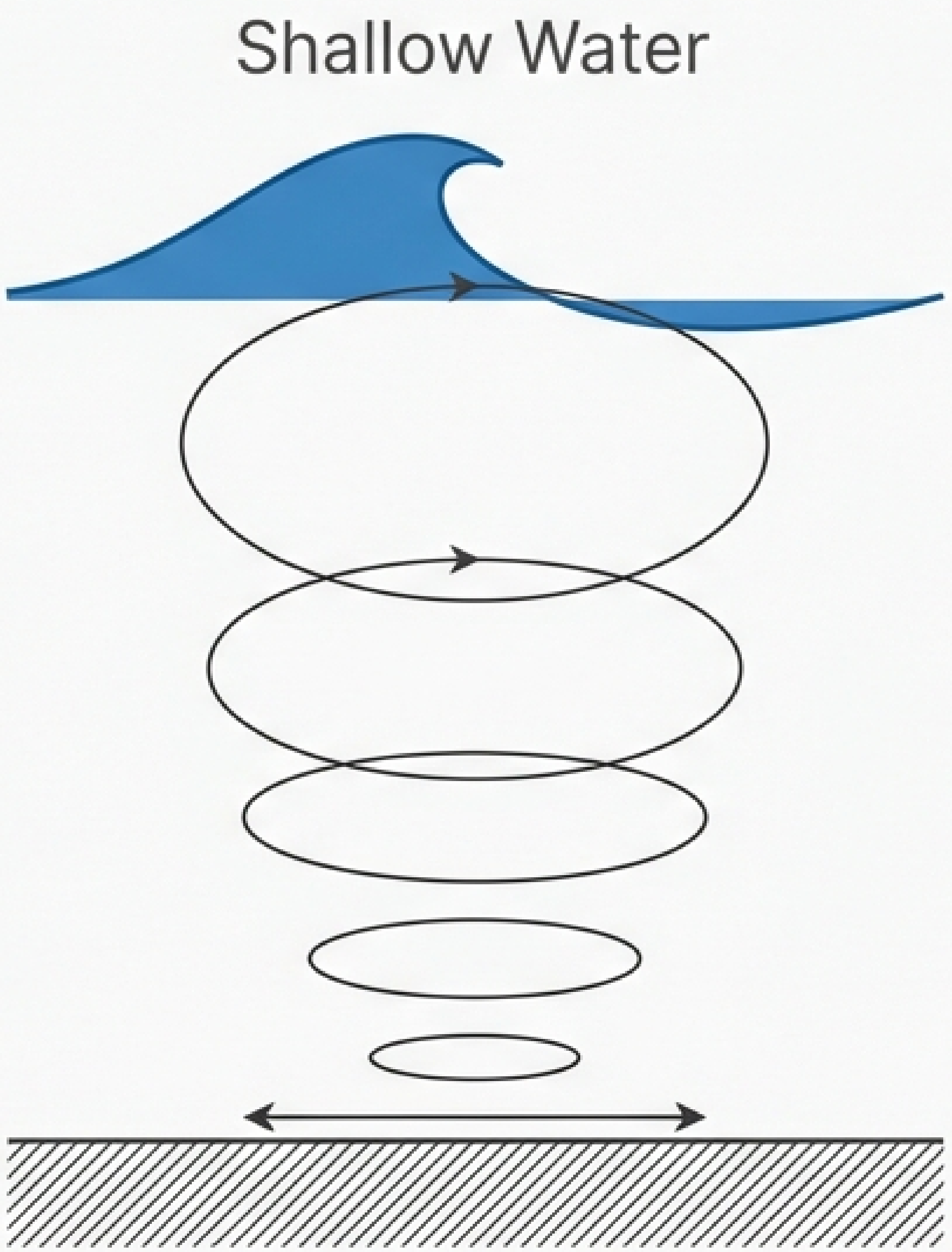



As a wave enters shallow water: Period ($T$) stays constant, Speed ($C$) decreases, Wavelength ($L$) decreases.

# Beneath the Surface: Particle Kinematics




Circular orbits. Motion decays to zero at depth $L/2$.




Elliptical orbits. Horizontal motion dominates near the bed.

Horizontal Velocity: $u$ →

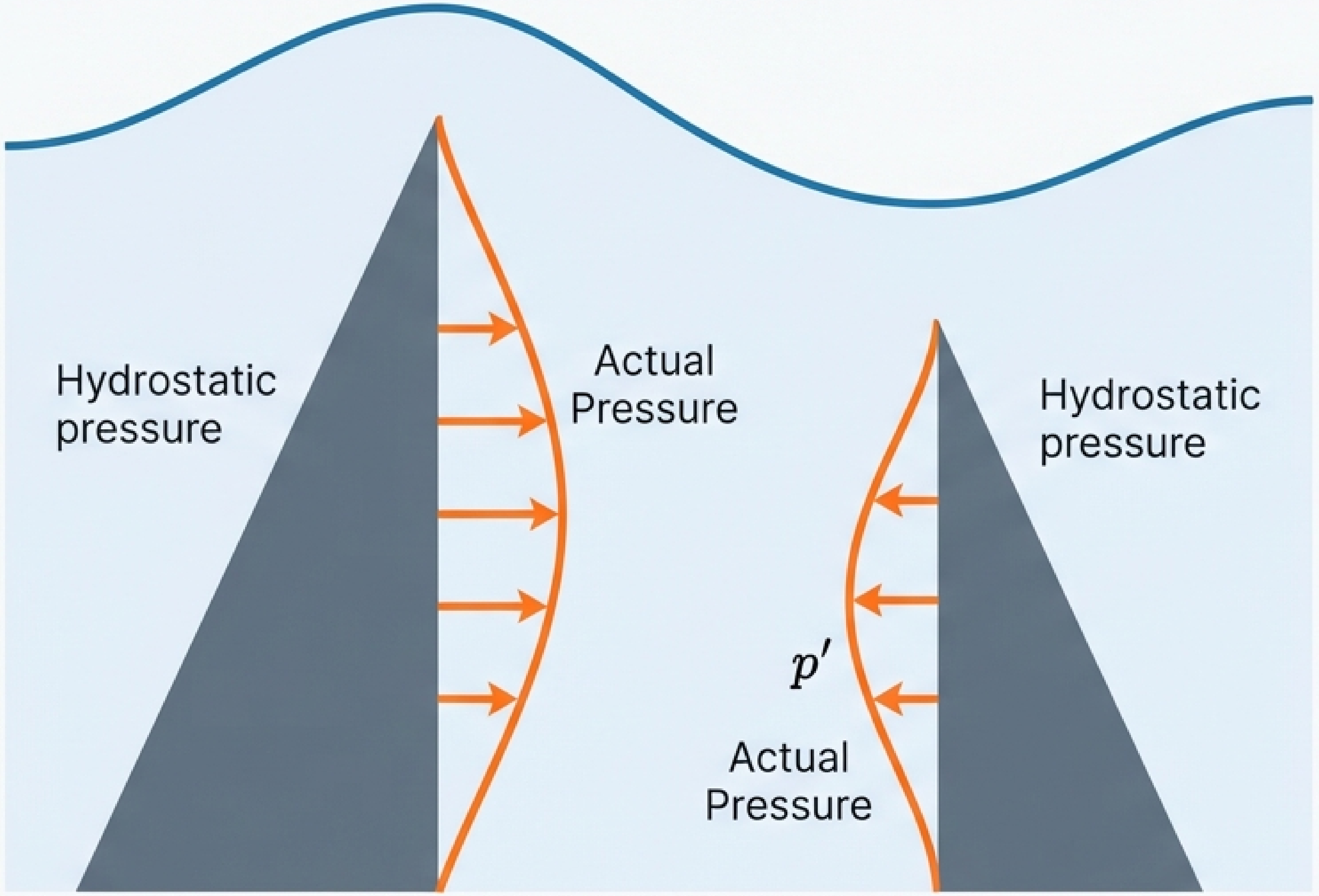
$$u = a\omega \frac{\cosh k(z+h)}{\sinh kh} \cos(kx - \omega t)$$

Vertical Velocity: $w$ ↕

$$w = a\omega \frac{\sinh k(z+h)}{\sinh kh} \sin(kx - \omega t)$$

NotebookLM

# Pressure Fields and Depth Decay




Key Concept

Total Pressure ($P$) = Static ($P_{static}$) + Dynamic ($p'$)

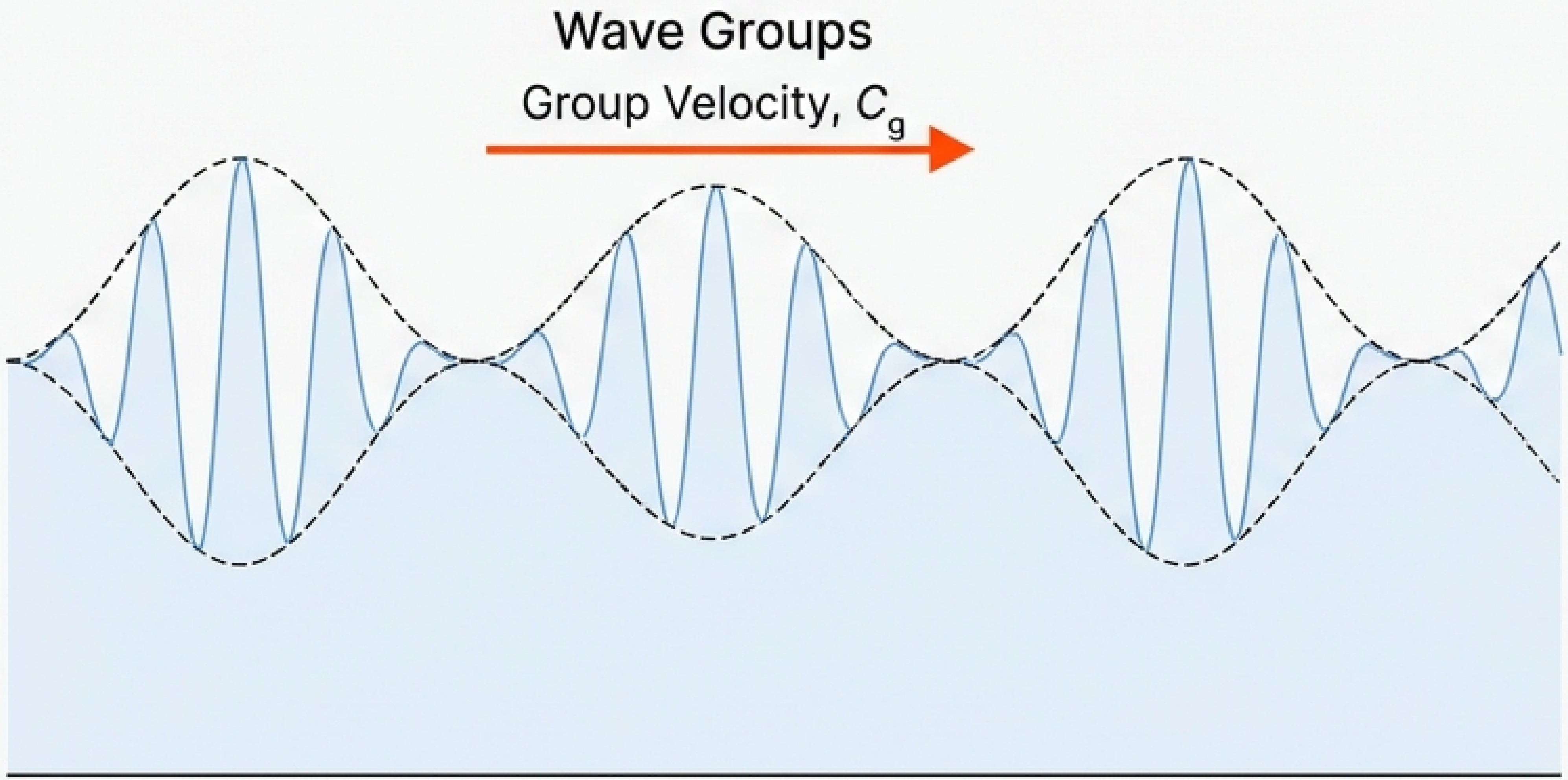
$$p' = \rho g a \frac{\cosh k(z+h)}{\cosh kh} \cos(kx - \omega t)$$

**Insight**

Dynamic pressure decays exponentially. A submarine deep enough experiences constant static pressure, unaffected by the storm waves above.

# Energy Density and Group Velocity

Energy travels at a different speed than the waves themselves.




Energy Density

$$E = \frac{1}{8}\rho g H^2$$

(Note: Depends on height squared)

Group Velocity ($C_g$)

Deep Water:

$$C_g = \frac{1}{2}C$$

(Energy moves at half speed)

Shallow Water:

$$C_g = C$$

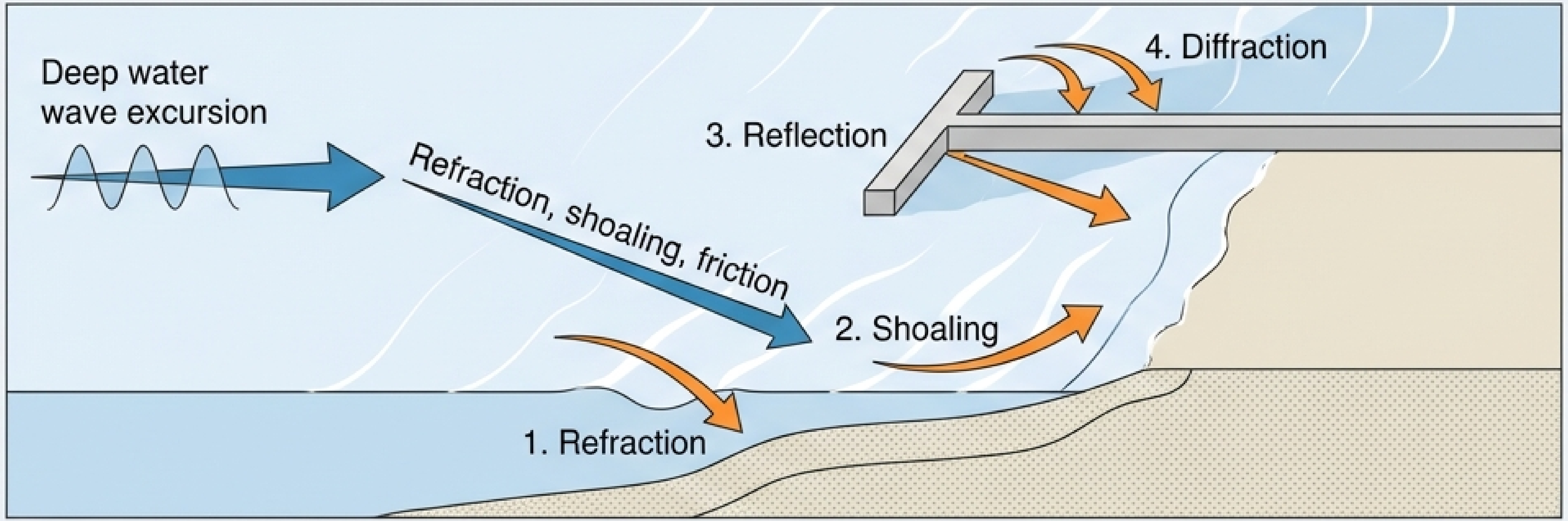
(Energy moves at full speed)

Power ($P$) is the Energy Flux: $P = EC_g$

# Comparative Analysis: Wave vs. Tidal Energy

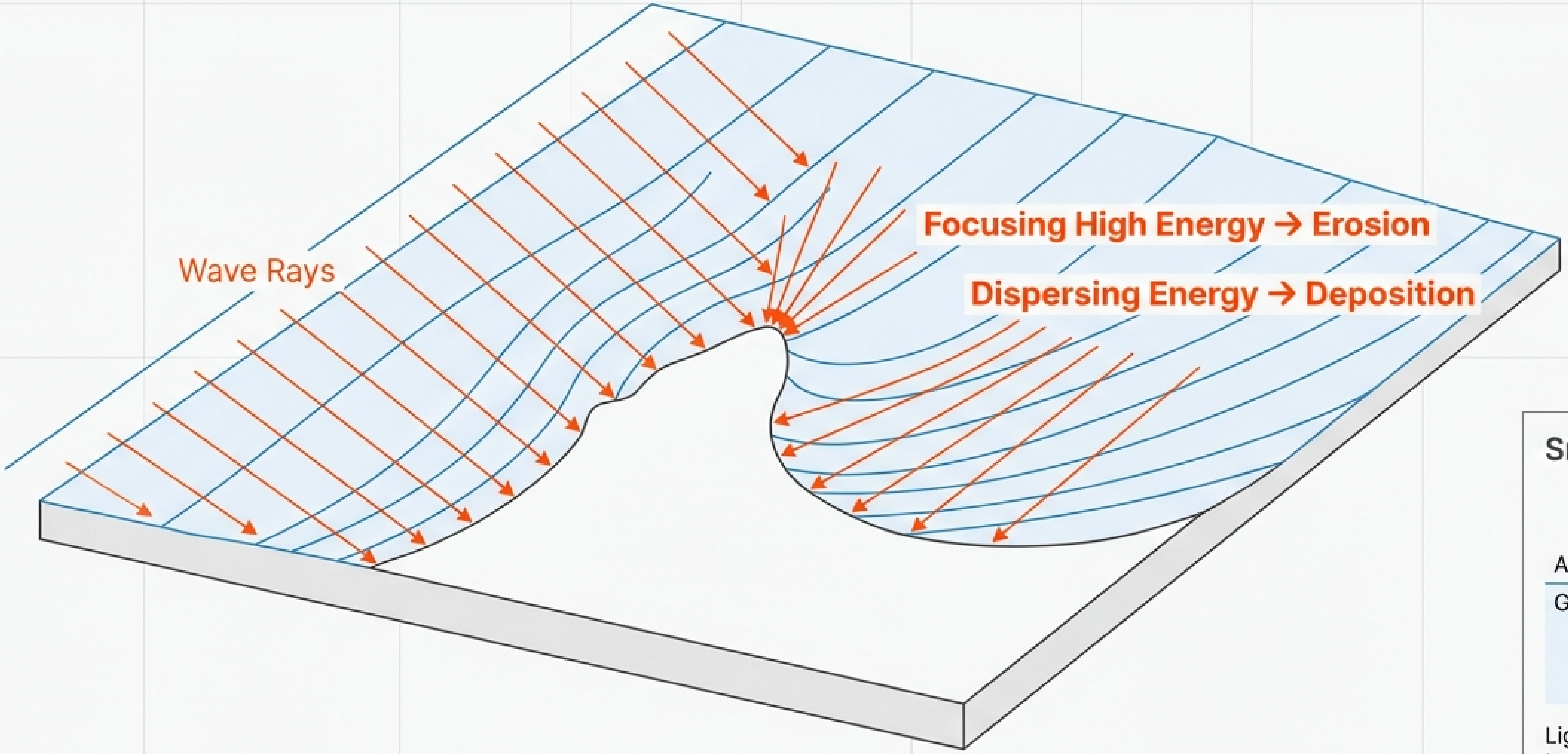
| Wave Energy | Tidal Energy |
|---|---|
| **Source:** Wind-driven (Atmospheric transfer). | **Source:** Gravitational (Moon/Sun). |
| **Predictability:** Irregular, Storm-dependent. | **Predictability:** Highly Deterministic (Astronomical cycles). |
| **Period:** High frequency (5–20 seconds). | **Period:** Low frequency (12.42 or 24.84 hours). |
| **Energy:** Kinetic + Potential (Surface oscillation). | **Energy:** Potential (Head difference) + Kinetic (Currents). |

# Wave Transformation: The Encounter with Land

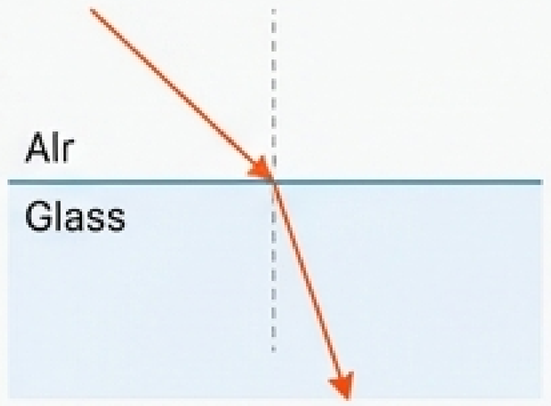



As the wave 'feels' the bottom, the governing physics shift from deep-water independence to bathymetric control.

# Refraction: The Bending Wave




### Snell's Law Analogy

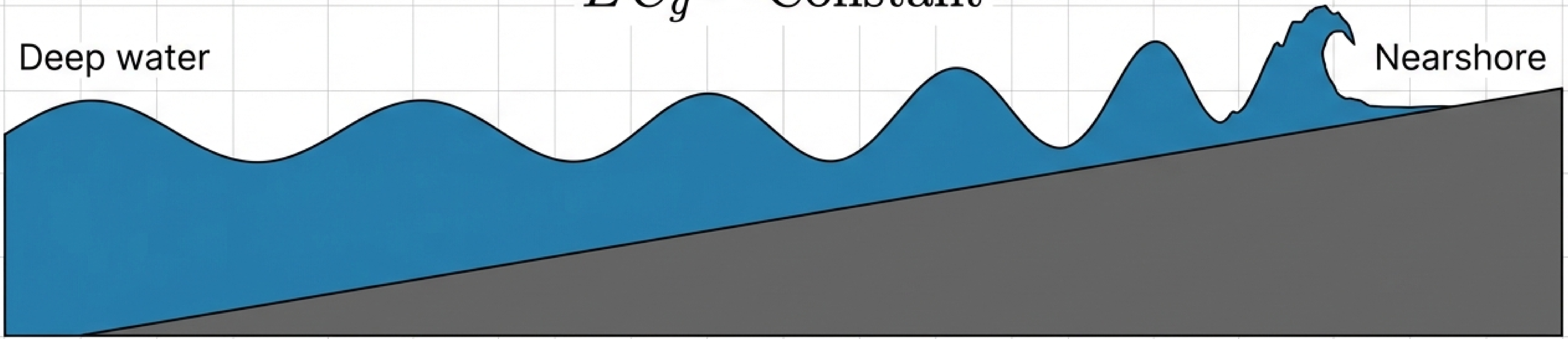



Light ray bends at interface between two media.

Just as light bends in glass, waves bend in shallow water because the inshore section slows down first.

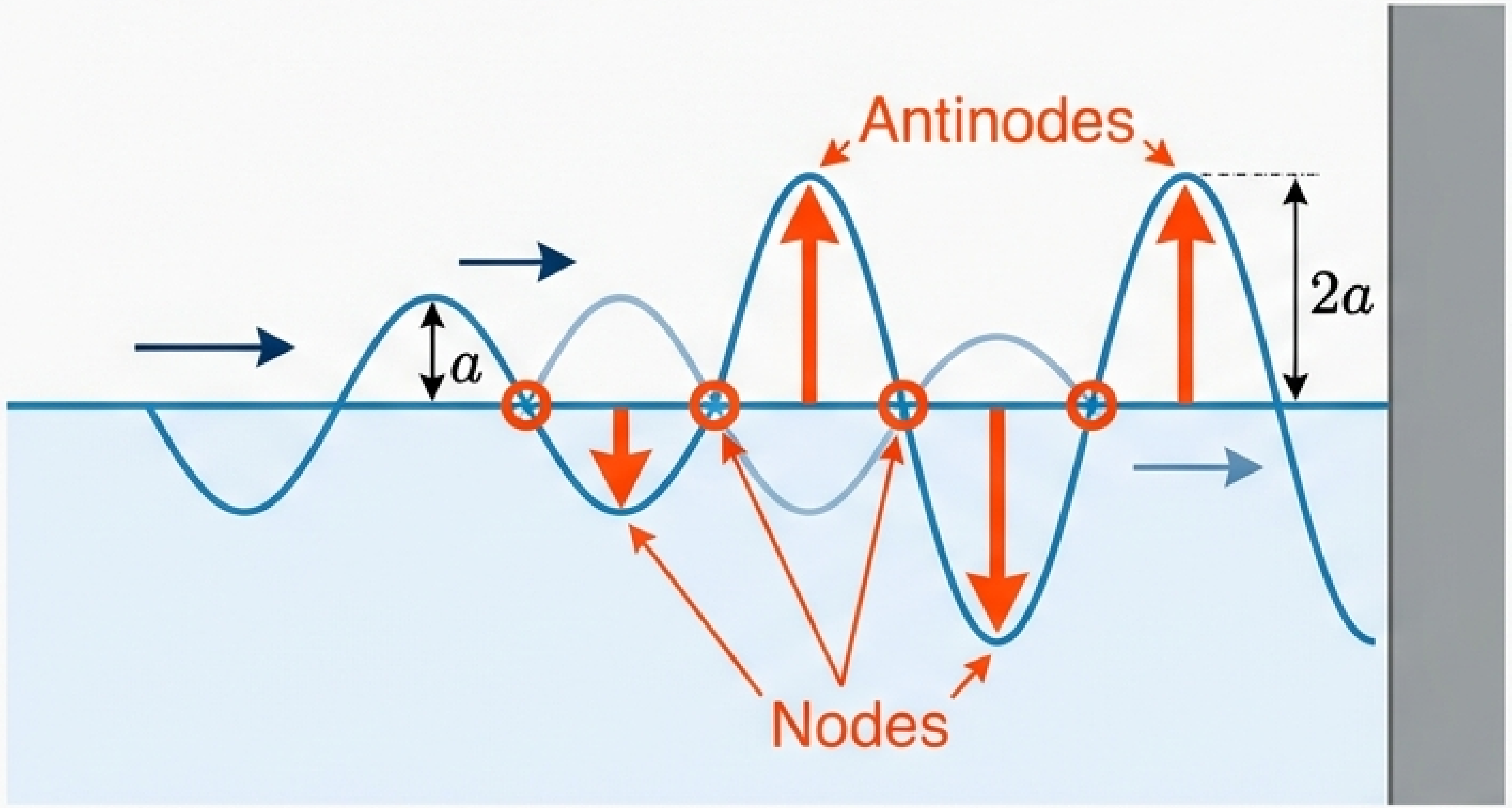
# Shoaling: Conservation of Energy Flux

$$E\,C_g = \text{Constant}$$




1. Depth decreases → 2. Group Velocity ($C_g$) slows down → 3. Energy Density ($E$) must increase to conserve flux → 4. Wave Height ($H$) increases

# Reflection and Standing Waves

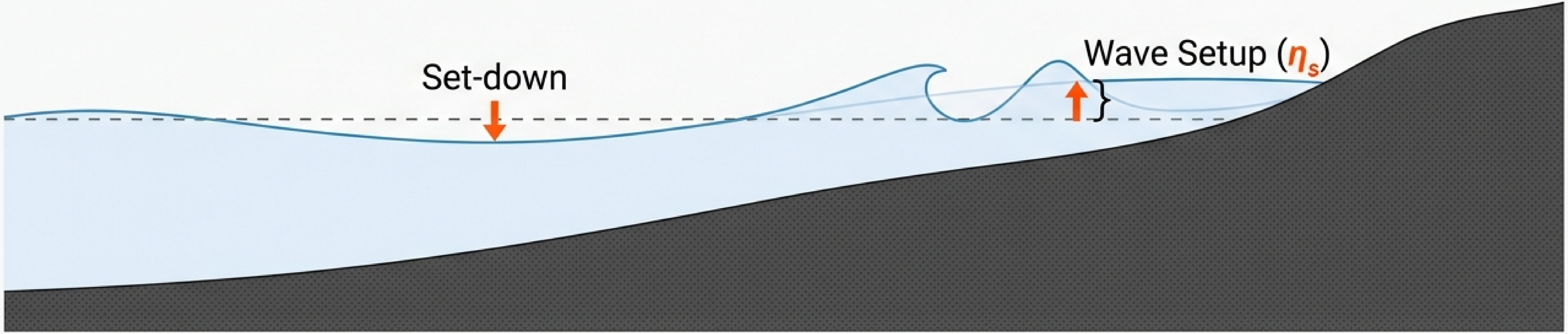



### Reflection Coefficients ($K_r$)

| Boundary Type | Reflection Coefficient $K_r$ | Typical Behavior |
|---|---|---|
| Vertical Wall / Cliff | $K_r \approx 1.0$ | Total Reflection |
| Rubble Mound | $K_r \approx 0.5$ | Partial Absorption |
| Sandy Beach | $K_r \approx 0.05$ | Total Absorption/ Dissipation |

# Radiation Stress and Wave Setup

Radiation Stress ($S_{xx}$): The excess flow of momentum due to wave presence.

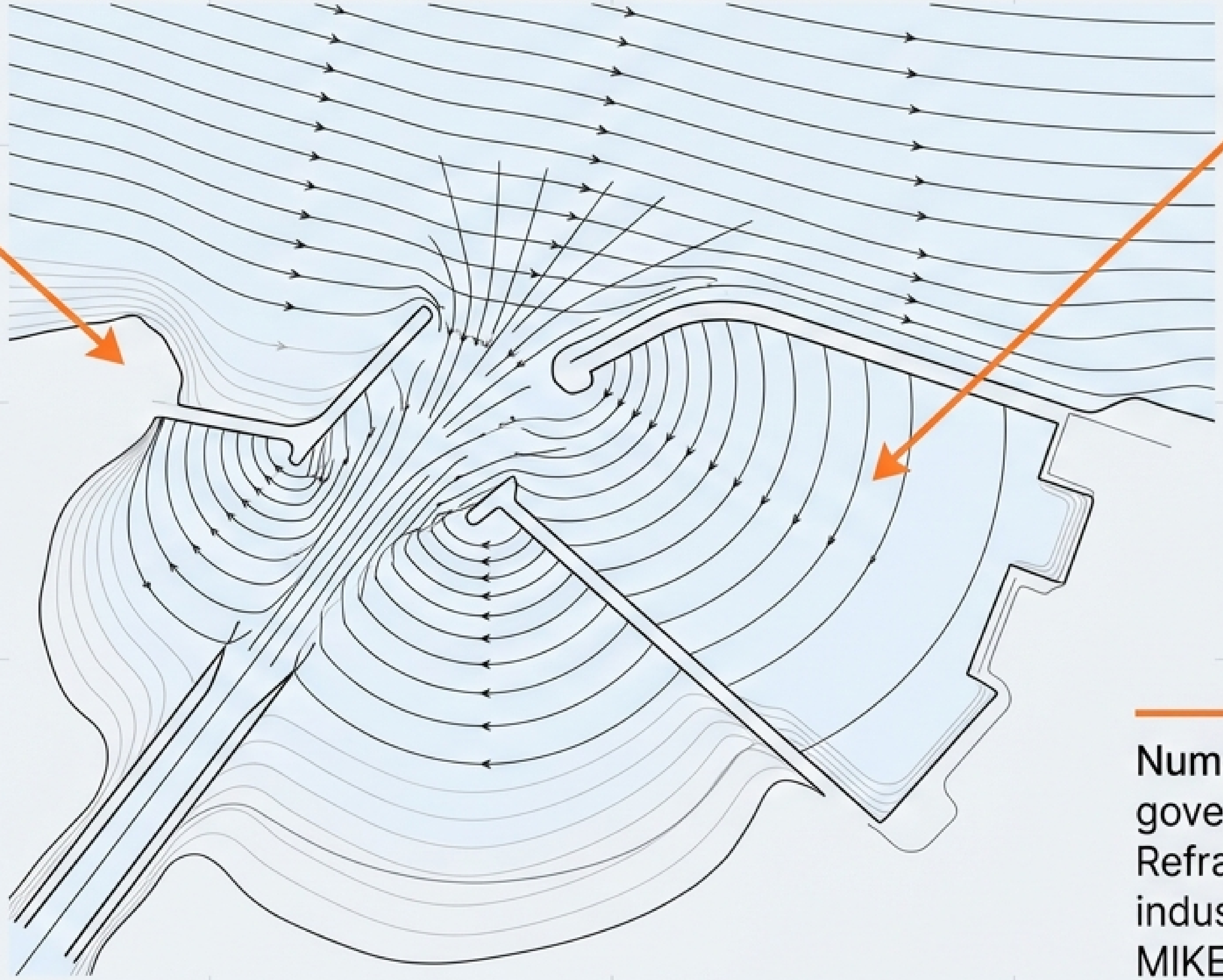



$$\frac{dS_{xx}}{dx} = \rho g h \frac{d\eta_s}{dx}$$

**Impact Note:** Setup increases coastal flooding levels during storms, allowing **waves to attack higher up the dunes**.

# Engineering Implications

Coastal Defense: Understanding refraction identifies erosion hotspots (Headlands) vs. safe zones (Bays).



Infrastructure Design: Calculating reflection prevents destructive resonance in ports.

**Numerical Modeling:** These governing laws ($S_{xx}$, Flux, Refraction) are the core of industry models like SWAN and MIKE21 used for flood prediction.